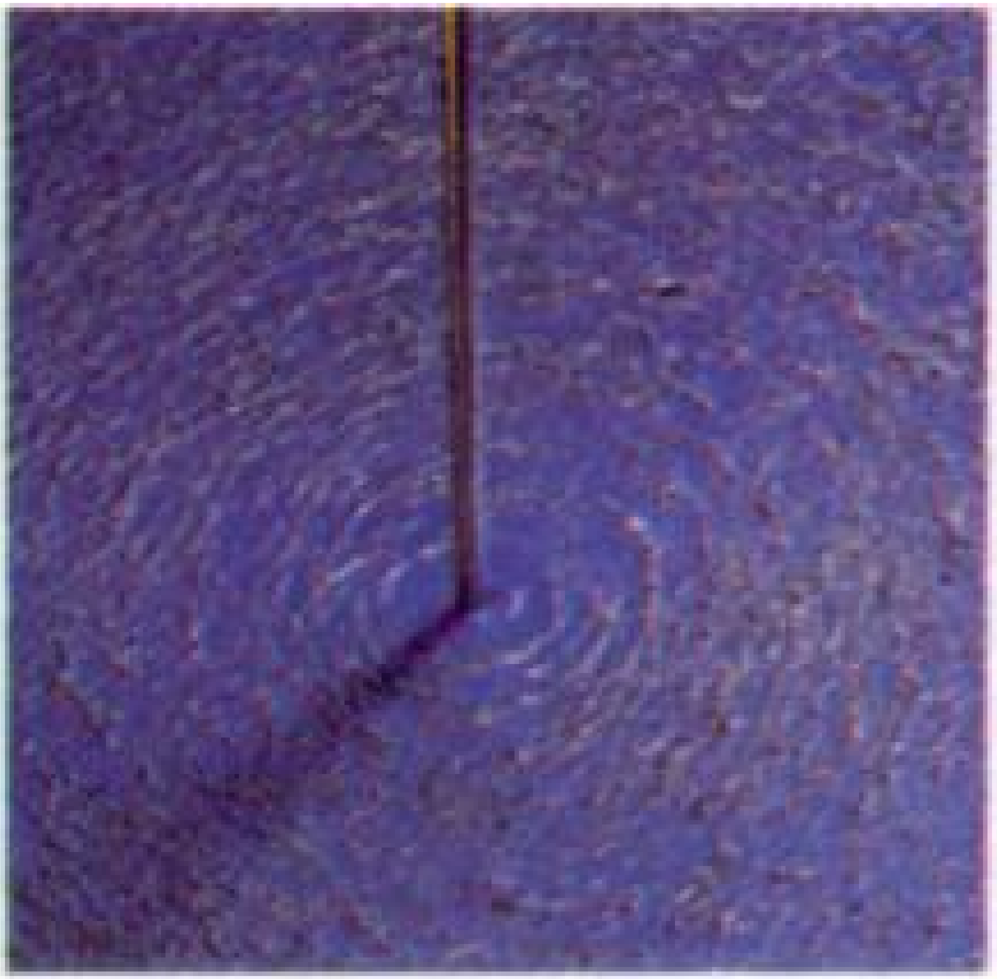
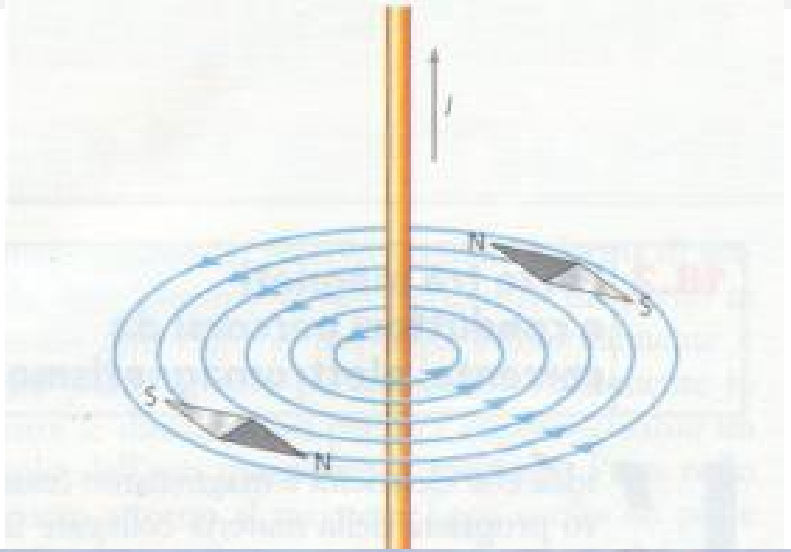
Magnetic imaging

Magnetic imaging

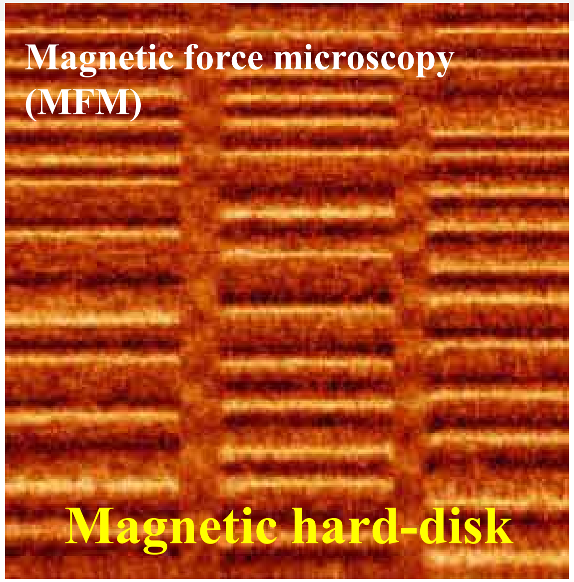
- Introduction
- Magnetic Force Microscopy (MFM)
- Scanning Hall Probe Microscopy (SHPM)
- Magnetic Resonance Imaging (MRI)
- Other techniques

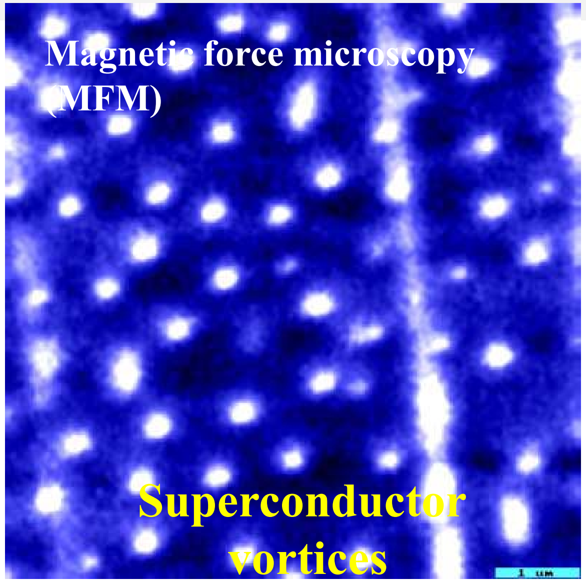
Magnetic imaging: «preliminary» experiments….

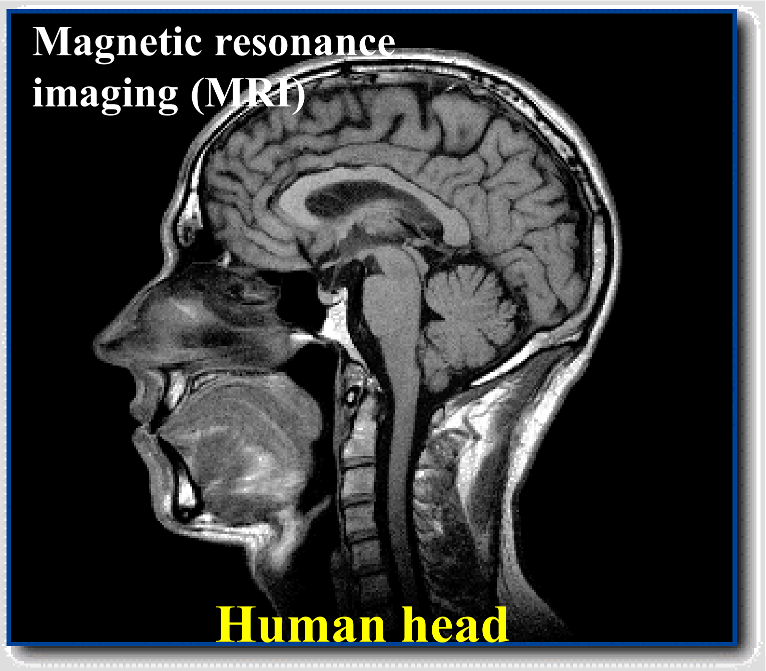


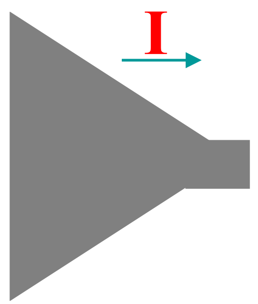


Magnetic imaging: a powerful tool….

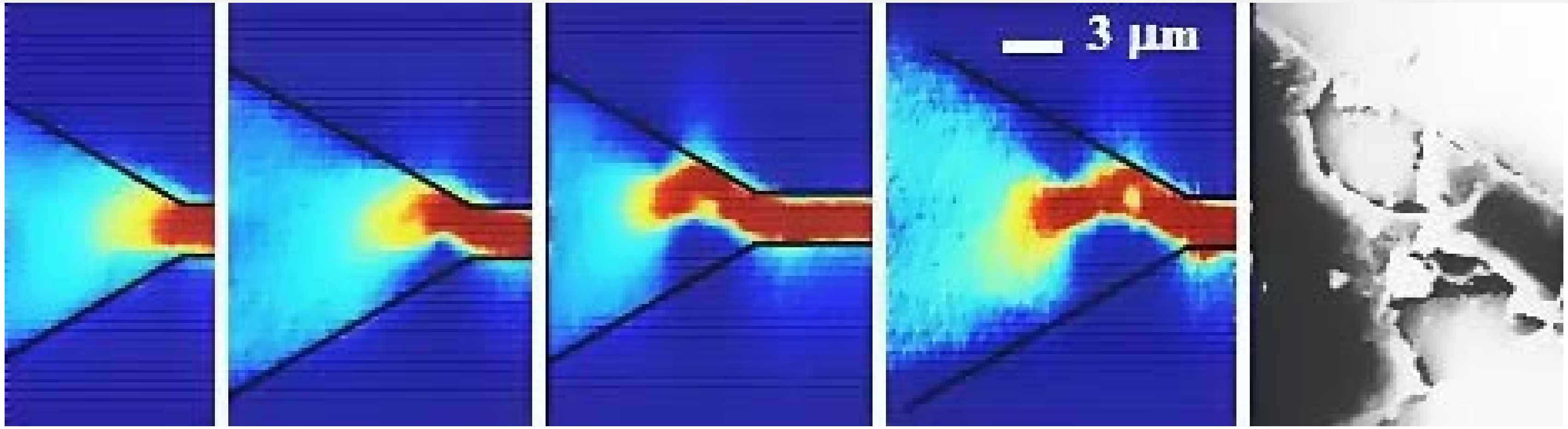


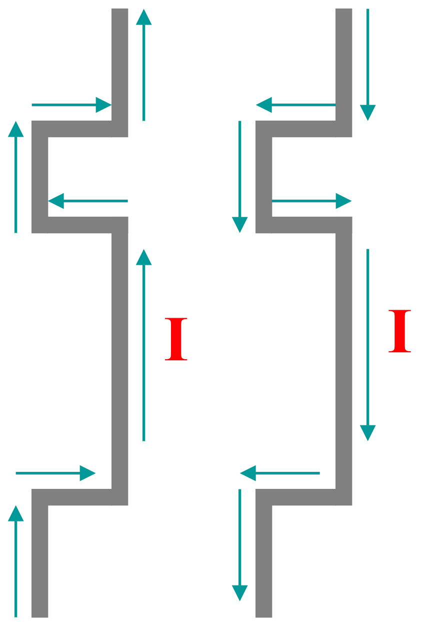




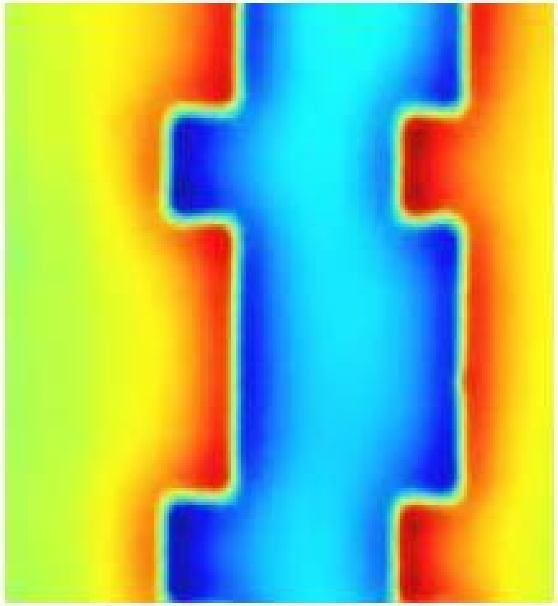


Wire with constriction

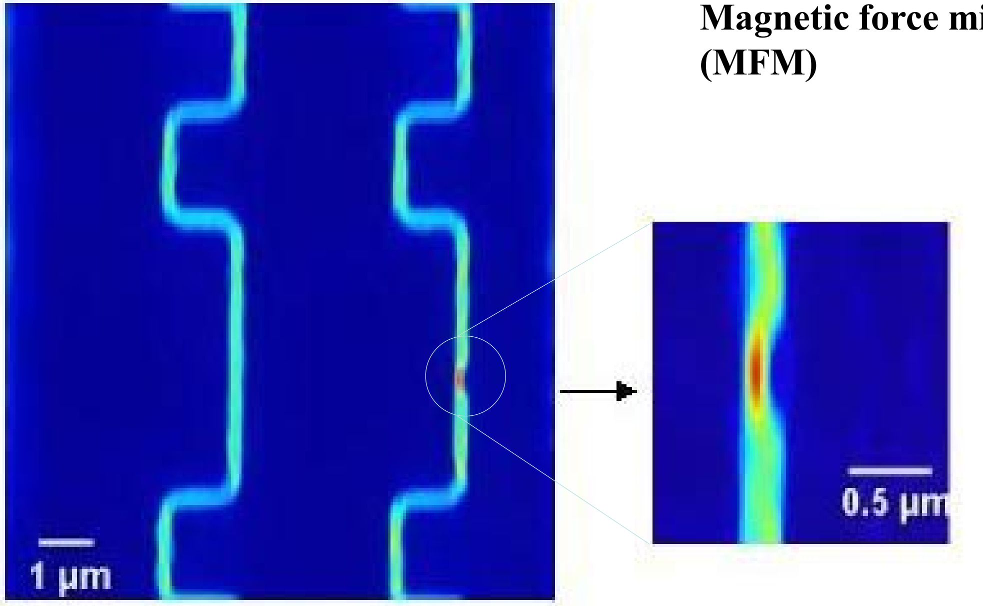




Two wires (2.5 µm wide)



Field Image



Current Image

Magnetic force microscopy (MFM)

Magnetic fields: the microscopic origin…

Magnetic fields sources:

a) macroscopic motion of charged particle (electrons in a wire, ions in a beam, …)

b) microscopic 'motion' of charged particle (electrons in a atom, …)

c) intrinsic magnetic moment of some particle (electrons, neutrons, some nuclei,…)

Electron

$$\mu_e \cong \mu_B \cong 9.3\times10^{-24}\,\mathrm{Am}^2$$

Bohr magneton $\mu_B = \dfrac{e\hbar}{2m_e}$

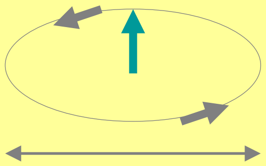
Proton (^{1}H nucleus)

$$\mu_p \cong 2.8\times\mu_N \cong 1.4\times10^{-26}\,\mathrm{Am}^2$$

Nuclear magneton $\mu_N = \dfrac{e\hbar}{2m_p}$

Current in a ring

1 A



1 cm

$$\mu \cong 10^{-4}\,\mathrm{Am}^2$$

Magnetic imaging: basics…

Lorentz force

$$\mathbf{F} = e(\mathbf{E} + \mathbf{v} \times \mathbf{B})$$

→

Force on a magnetic moment in a magnetic field

$$\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$$

Maxwell equations

$$\nabla \times \mathbf{E} = -\frac{\partial \mathbf{B}}{\partial t}$$

→

Voltage induced at the coil ends

$$V = -\frac{\partial \Phi_{\mathbf{B}}}{\partial t}$$

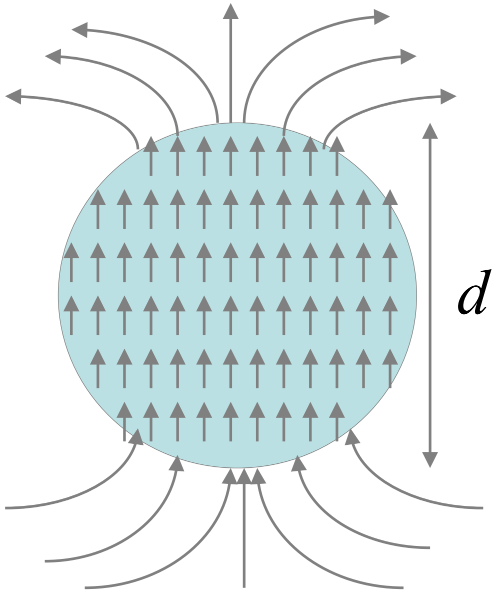
…………

…………

Quantum effects

………… → …………

Hard magnetic sphere



$\uparrow \approx \mu_B$

Sphere magnetization (A/m)

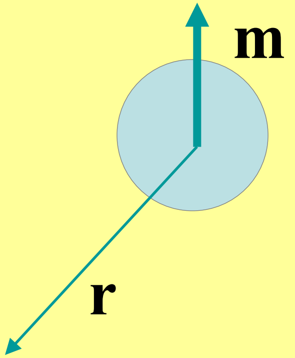
$$\mathrm{M} \approx \frac{N}{V}\mu_B$$

$$\frac{N}{V} \approx 10^{28}\,\text{atoms}/\text{m}^3$$

$$\mathrm{m} \approx N\mu_B$$

Atoms in the sphere

Sphere magnetic moment (Am^2)

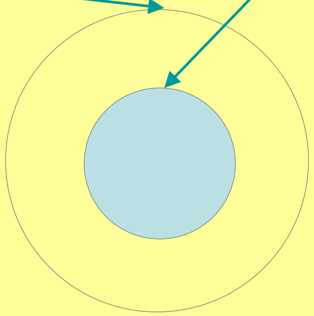


$$\hat{\mathbf{r}} \equiv \frac{\mathbf{r}}{r}$$

$$\mathbf{B}(\mathbf{r}) \cong \frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{m}) - \mathbf{m}}{r^3}\frac{\mu_0}{4\pi}$$

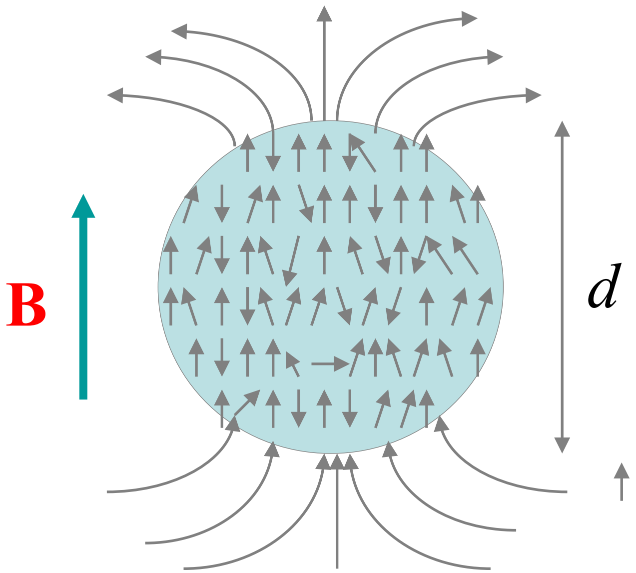
$$\mathrm{B}(0,0,d/2) \approx 100\ \text{mT}$$

$$\mathrm{B}(0,0,d) \approx 10\ \text{mT}$$



Sphere size independent

Paramagnetic sphere

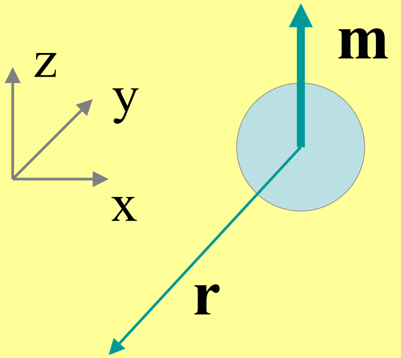


$\approx \mu_B$

For $\mu_B B_0 << k_B T$

$$M \approx \frac{N}{V}\mu_B\left(\frac{\mu_B B_0}{3kT}\right)$$

$$m \approx N\mu_B\left(\frac{\mu_B B_0}{3kT}\right)$$



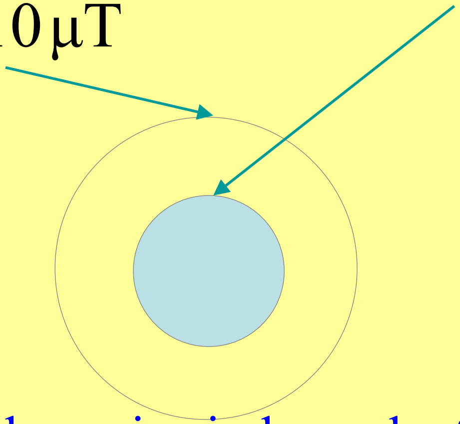
$r \equiv |\mathbf{r}|$

$\hat{\mathbf{r}} \equiv \frac{\mathbf{r}}{r}$

$$\mathbf{B}(\mathbf{r}) \cong \frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}}\cdot\mathbf{m})-\mathbf{m}}{r^3}\frac{\mu_0}{4\pi}$$

$B(0,0,d/2) \approx 100\,\mu T$

$B(0,0,d) \approx 10\,\mu T$

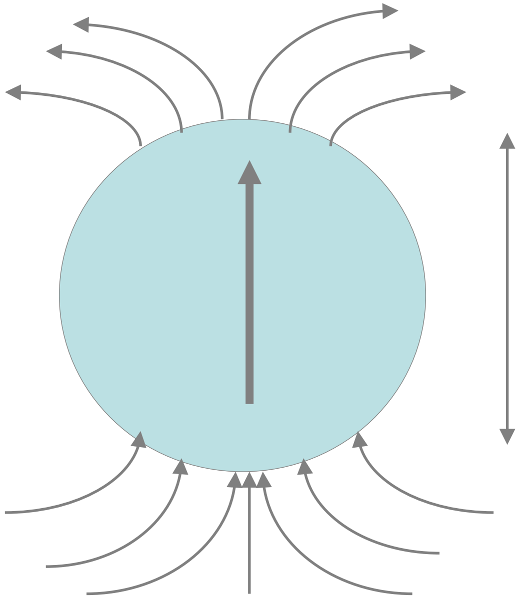


$B_0 = 1\,T$

$T = 300\,K$

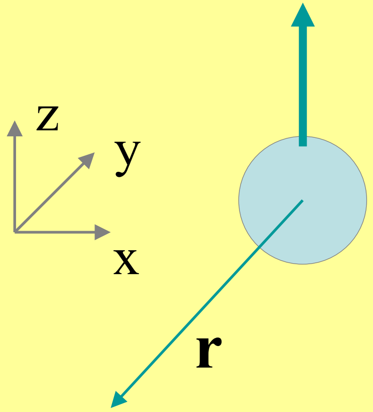
Sphere size independent

Single electron



$< 10^{-17}$ m

$$m \approx \mu_B$$



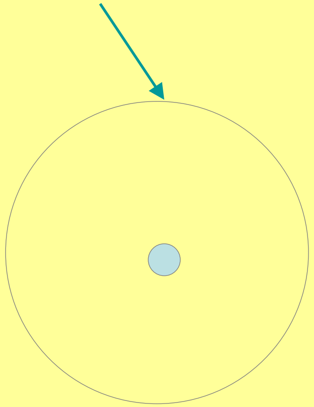
$$r \equiv |\mathbf{r}|$$

$$\hat{\mathbf{r}} \equiv \frac{\mathbf{r}}{r}$$

$$\mathbf{B}(\mathbf{r}) \cong \frac{3\hat{\mathbf{r}}(\hat{\mathbf{r}} \cdot \mathbf{m}) - \mathbf{m}}{r^3} \frac{\mu_0}{4\pi}$$

$$B(0,0,10\,\text{nm}) \approx 1\,\mu\text{T}$$

$$B(0,0,1\,\text{nm}) \approx 1\,\text{mT}$$

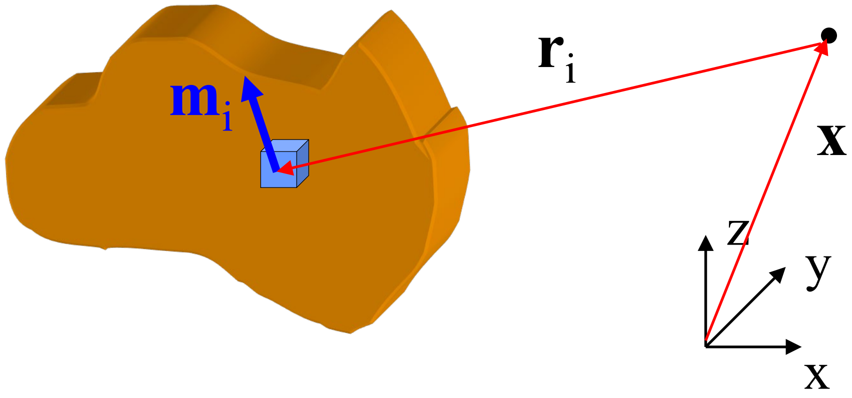


Electron size independent

Arbitrary object

$$r \equiv |\mathbf{r}|$$

$$\hat{\mathbf{r}} \equiv \frac{\mathbf{r}}{r}$$

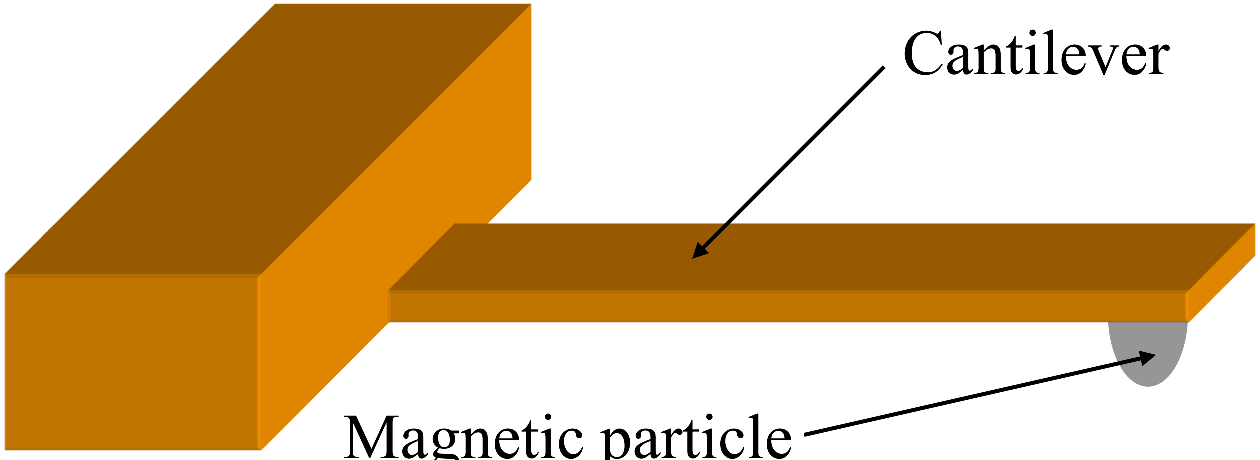


$$\mathbf{m}_i \cong \mathbf{M}(\mathbf{r}_i)\Delta V_i$$

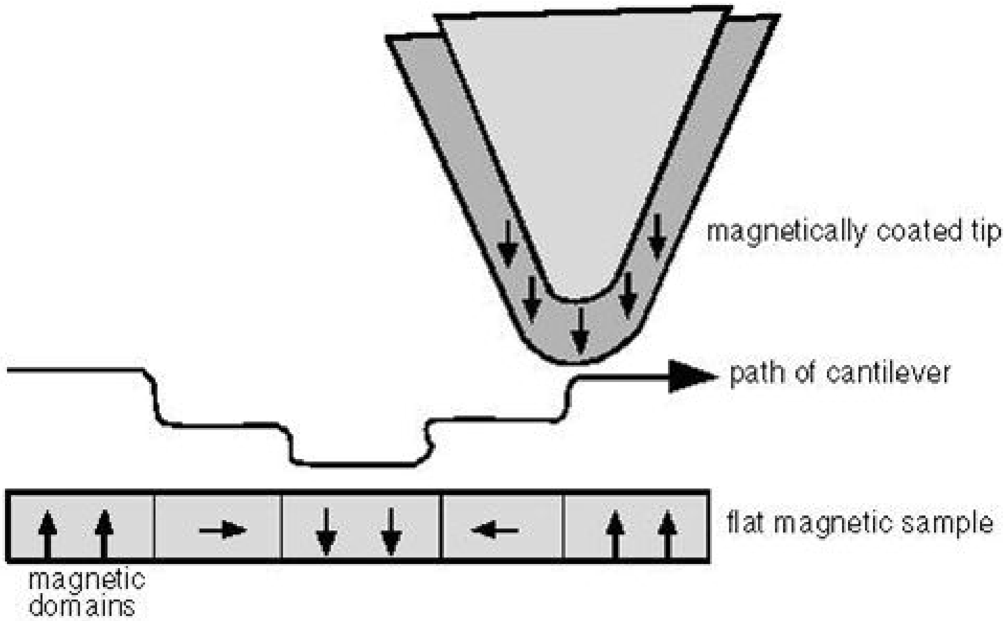
$$\mathbf{B}(\mathbf{x}) \cong \sum_i \frac{3\hat{\mathbf{r}}_i(\hat{\mathbf{r}}_i \cdot \mathbf{m}_i) - \mathbf{m}_i}{r_i^{\;3}} \frac{\mu_0}{4\pi}$$

Magnetic force microscopy (MFM)

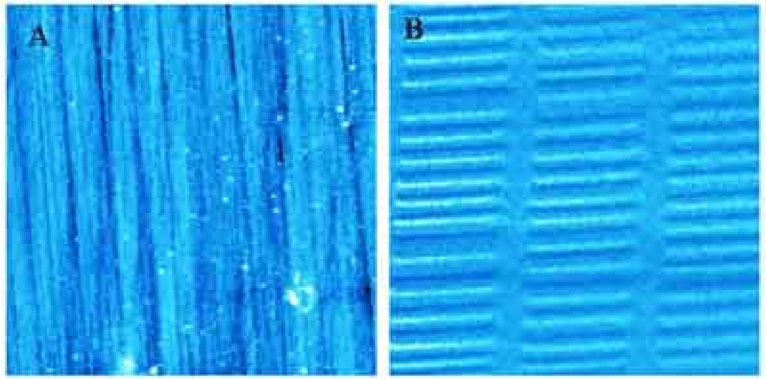
MFM: introduction



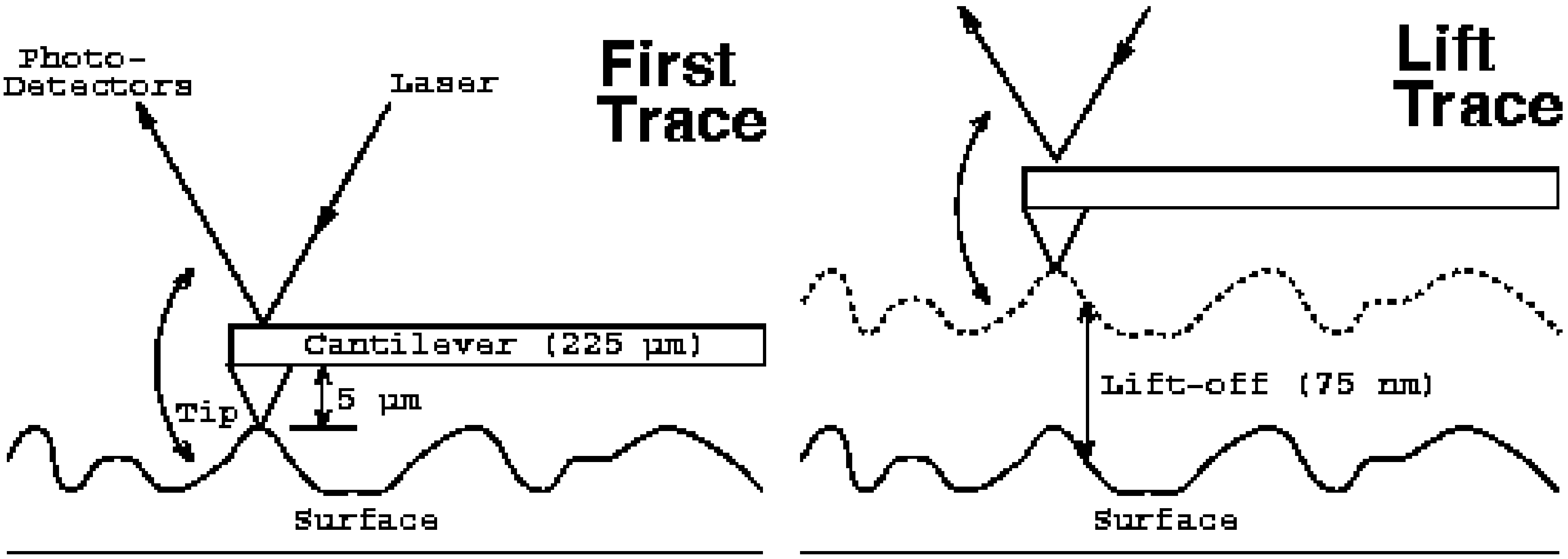
$$\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$$



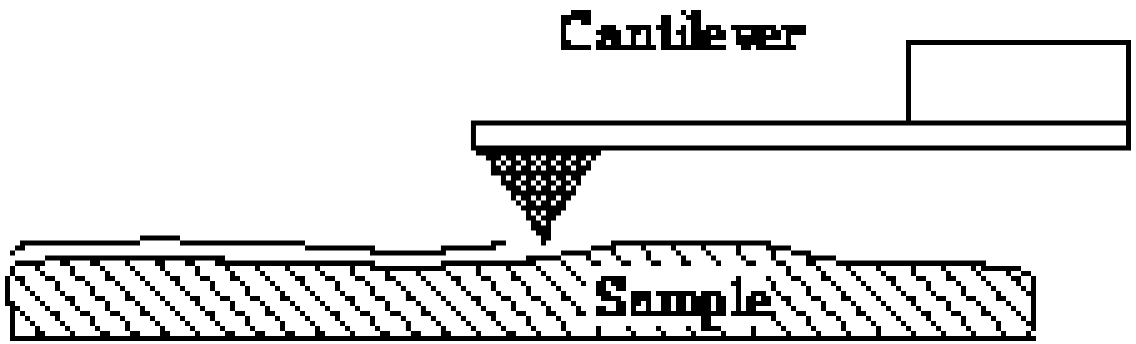
Topography **Magnetic structure**



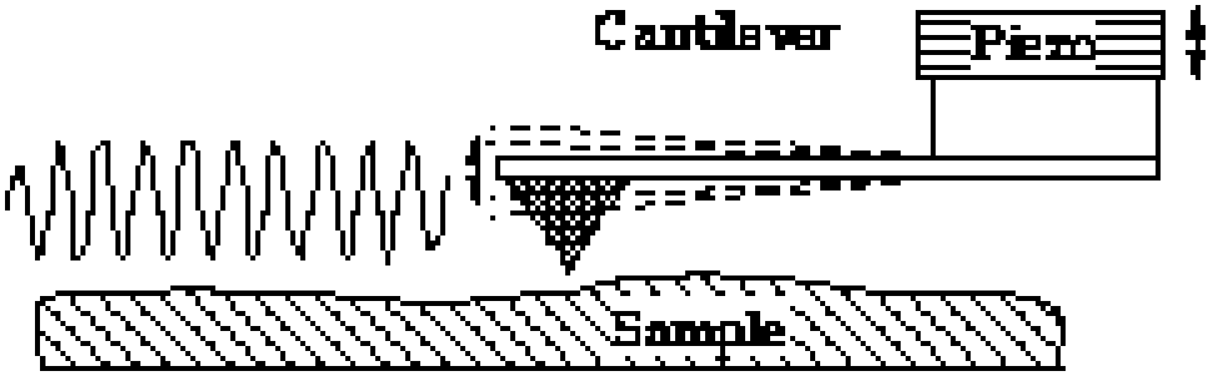
MFM: operating modes



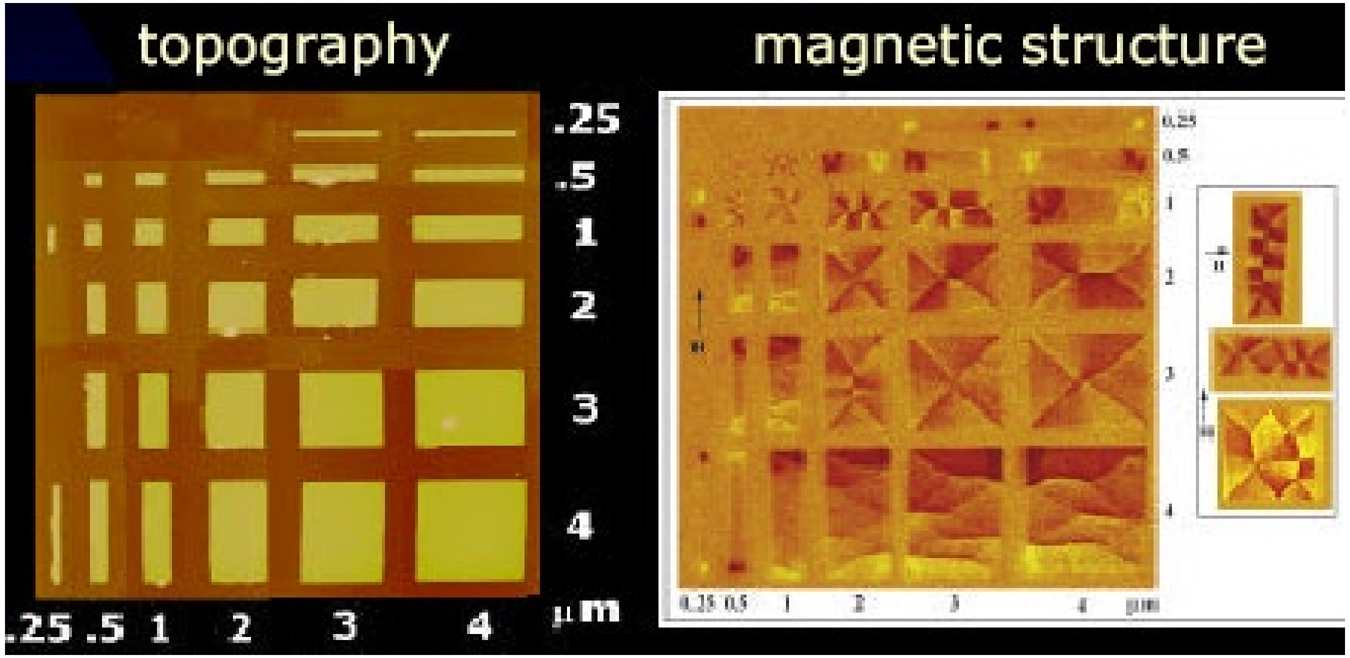
DC mode

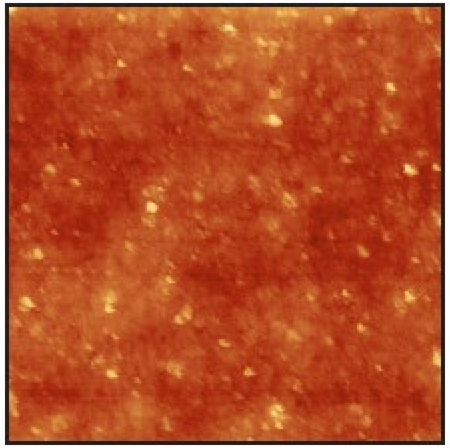


AC mode (at resonance)

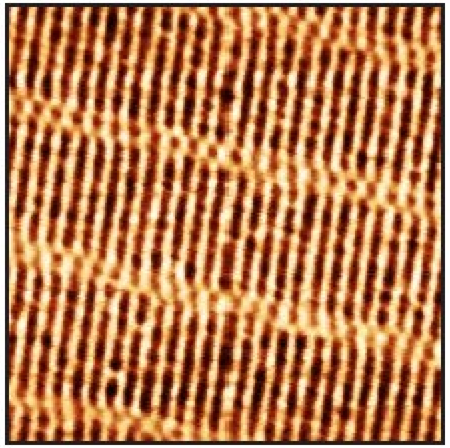


MFM: topography and magnetic imaging



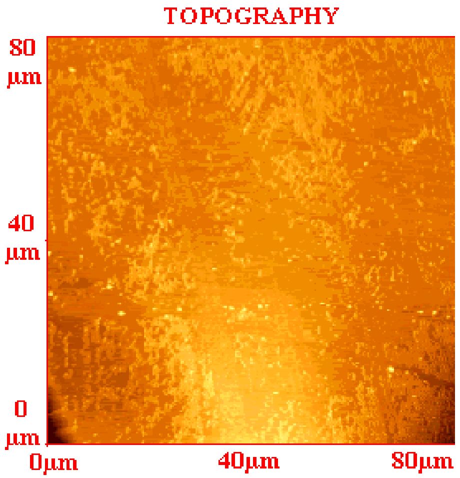


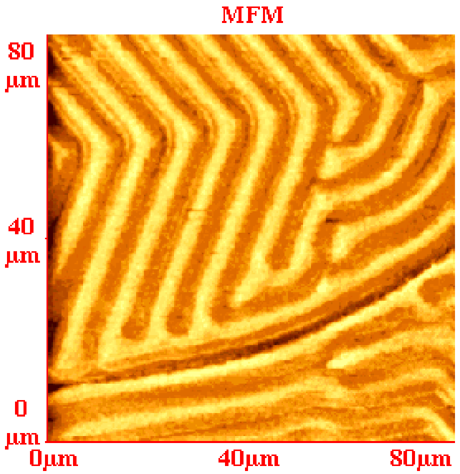
AFM image, Zip Disk 40 x 40 µm



MFM image, Zip Disk 40 x 40 µm

ZIP Disk





YIG film

MFM: key features

Strong points

- High spatial resolution (typical: $(100\ nm)^2$, best: $(20\ nm)^2$)
- Topographic and magnetic information

Weak points

- Invasive
- Hardly quantitative

Applications

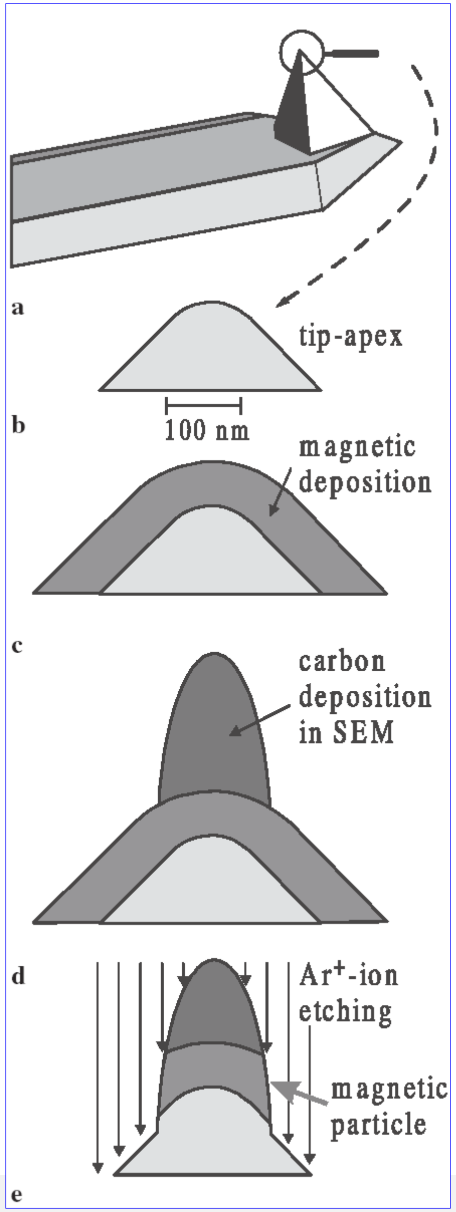
- Industry:

  High density magnetic memories development
- Research:

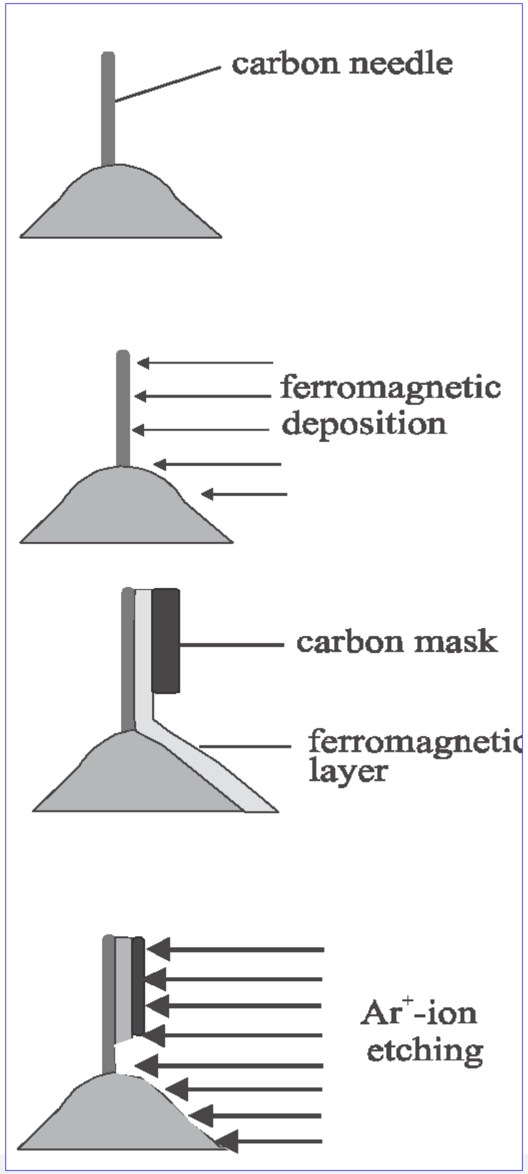
  magnetic materials, superconductors, magnetic bacteria, ….

MFM: magnetic tips fabrication

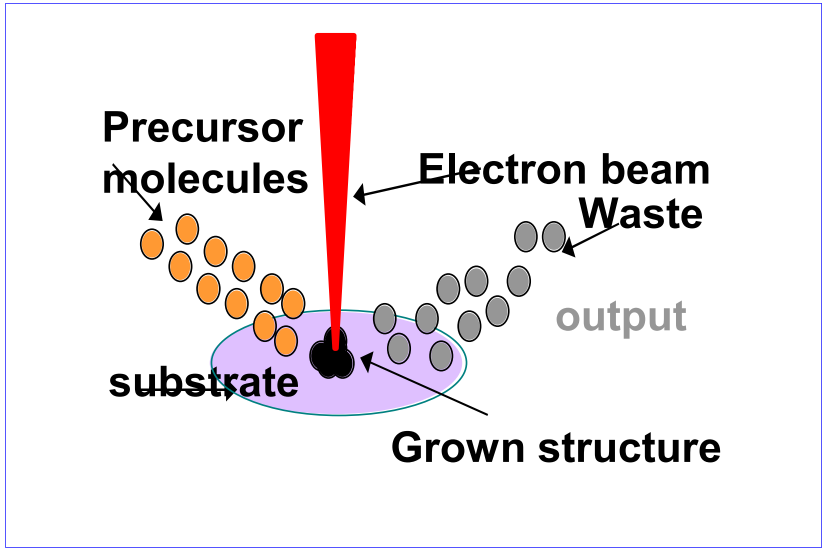
A) Longitudinal etching

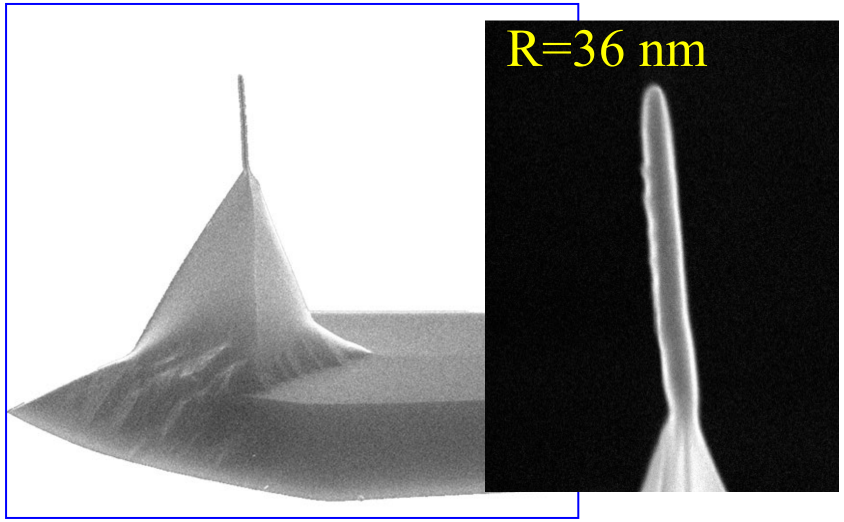


B) Lateral etching

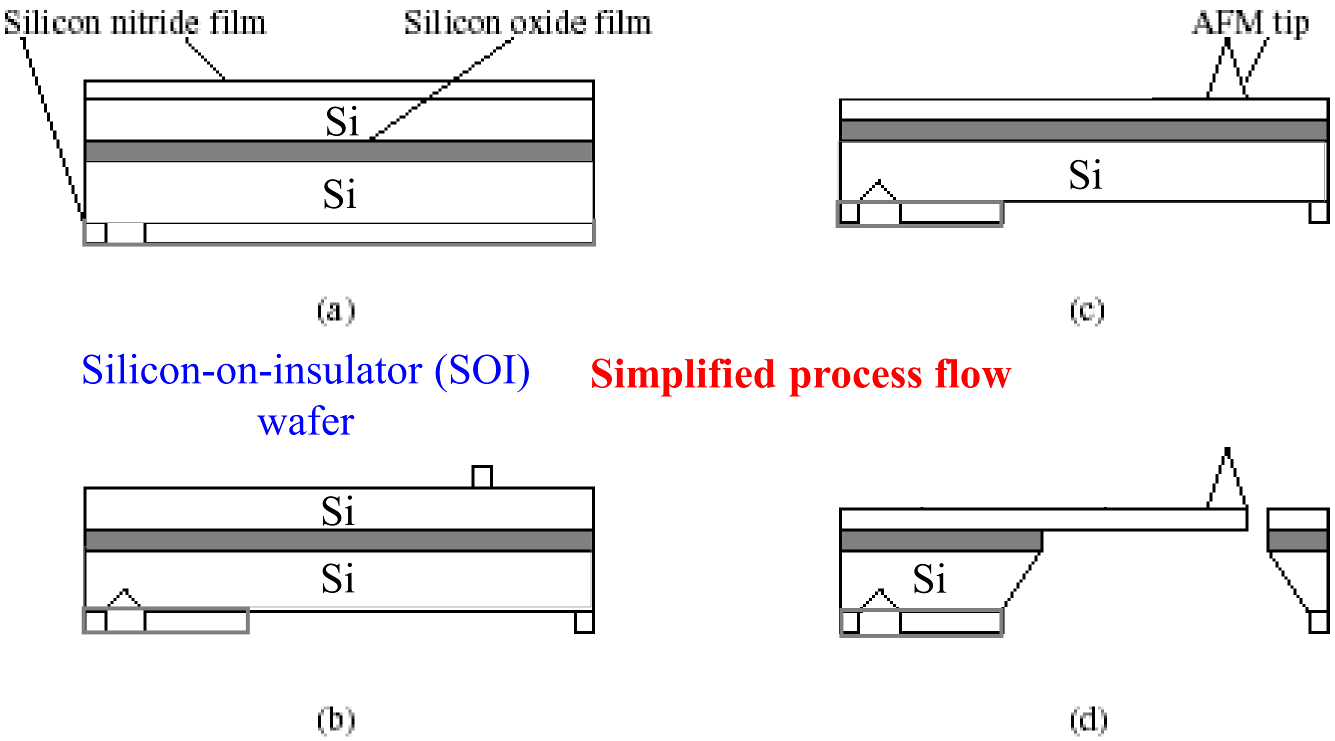


C) Electron beam induced deposition



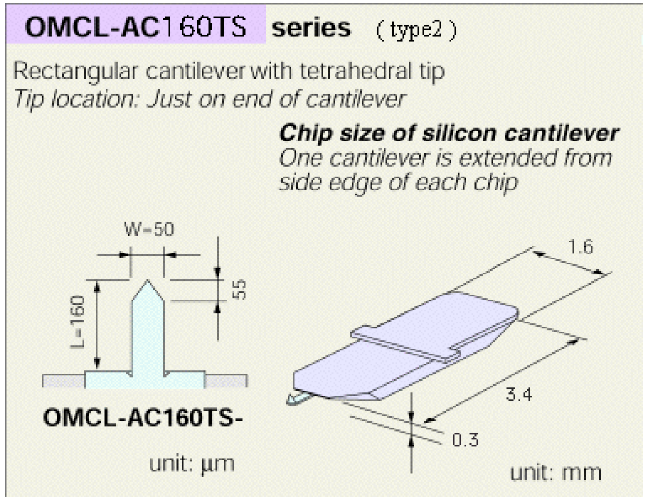


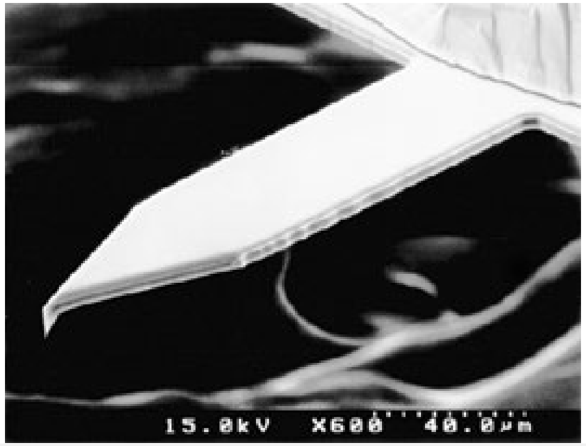
MFM: cantilevers fabrication

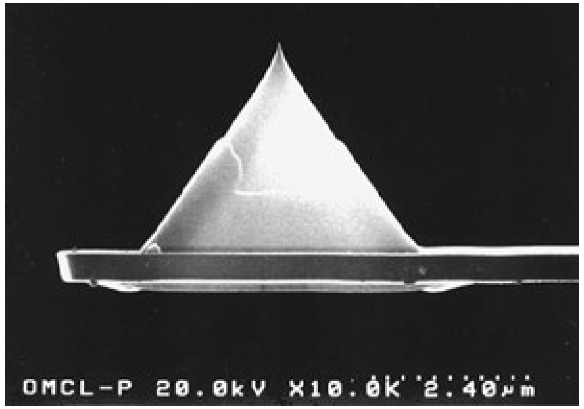


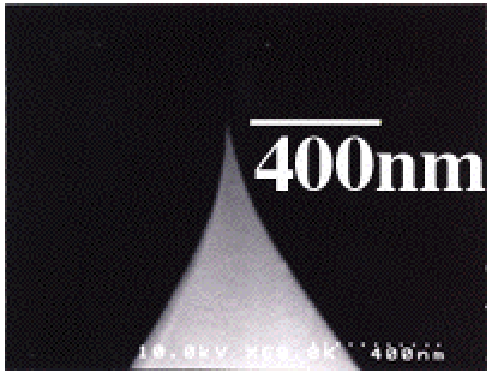
Silicon-on-insulator (SOI) wafer

Simplified process flow

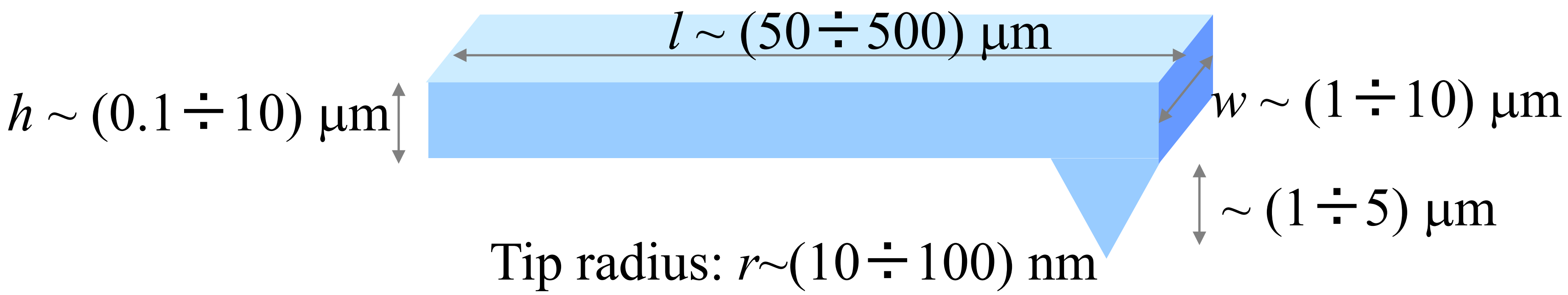








MFM: cantilever properties



$$k \cong \frac{Ew}{4}\left(\frac{h}{l}\right)^3$$

$k \sim (10^{-4} \div 10^{2})$ N/m

$$\omega_c \cong \sqrt{\frac{E}{\rho}}\frac{h}{l^2}$$

$\omega_c \sim (10^{2} \div 10^{8})$ Hz

E: Young modulus (N/m^2) (190x10^9 N/m^2 for Si)

k: spring constant (N/m)

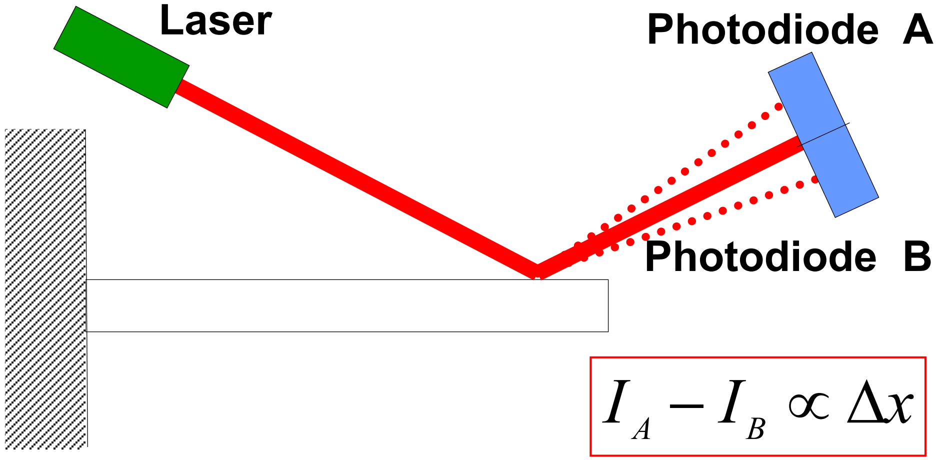
Q: Cantilever quality factor (unknown a priori)

ω_c: Cantilever resonance frequency (Hz)

ρ: Cantilever density (kg/m^3) (2333 Kg/m^3 for Si)

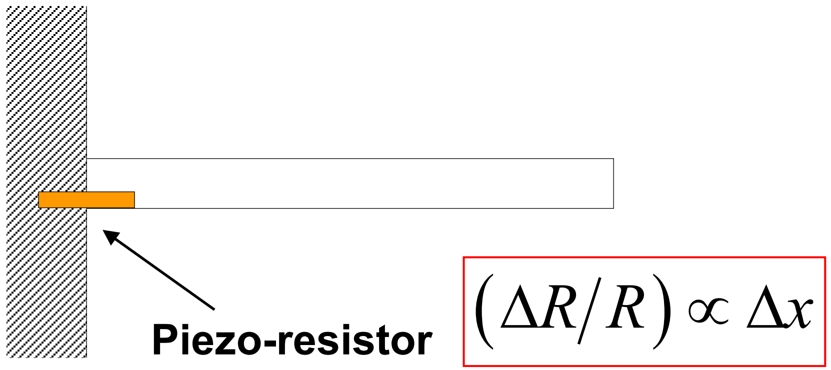
Cantilever displacement detectors

Laser beam deflection



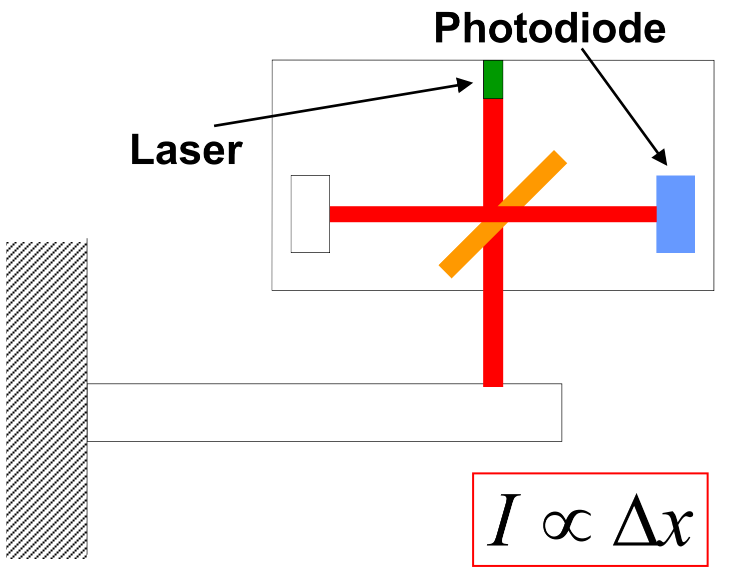
$$I_A - I_B \propto \Delta x$$

Piezo-resistive sensor



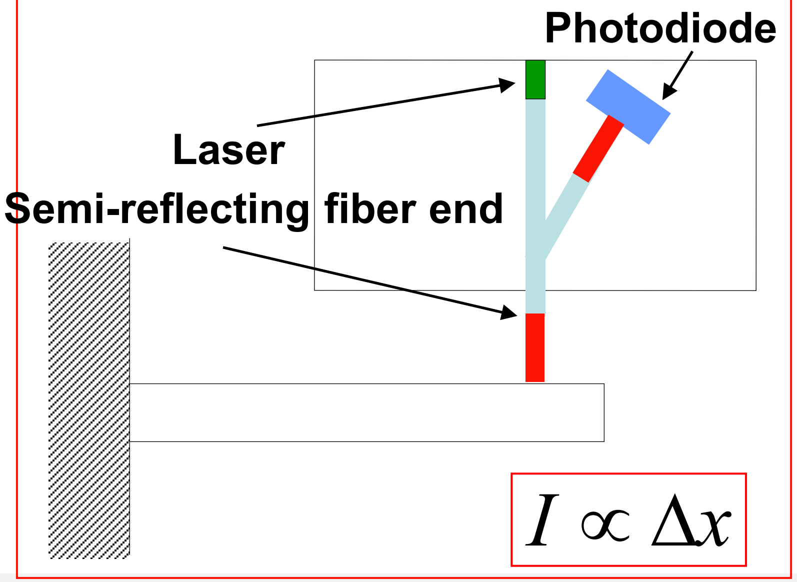
$$(\Delta R / R) \propto \Delta x$$

Laser interferometer (1)



$$I \propto \Delta x$$

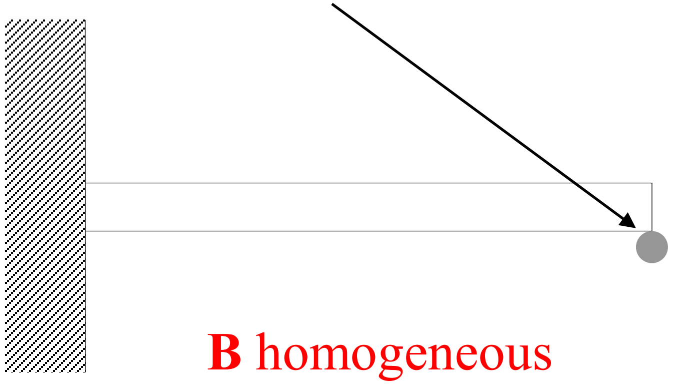
Laser interferometer (2)



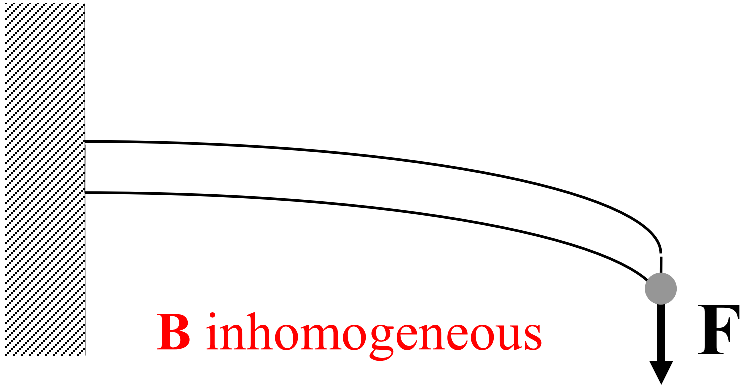
$$I \propto \Delta x$$

MFM: basic theory

Ferromagnetic particle
(total magnetic moment **m**)

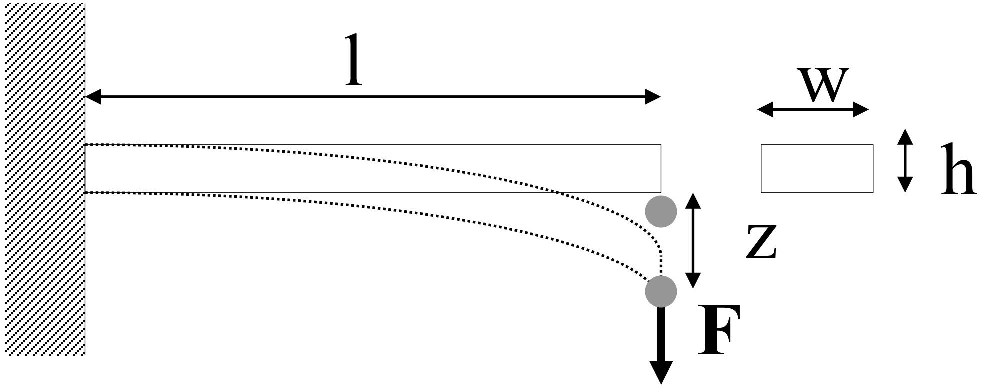


B homogeneous



B inhomogeneous

$$\mathbf{F} = \nabla(\mathbf{m} \cdot \mathbf{B})$$



$$\Delta z \cong \frac{1}{k} F$$

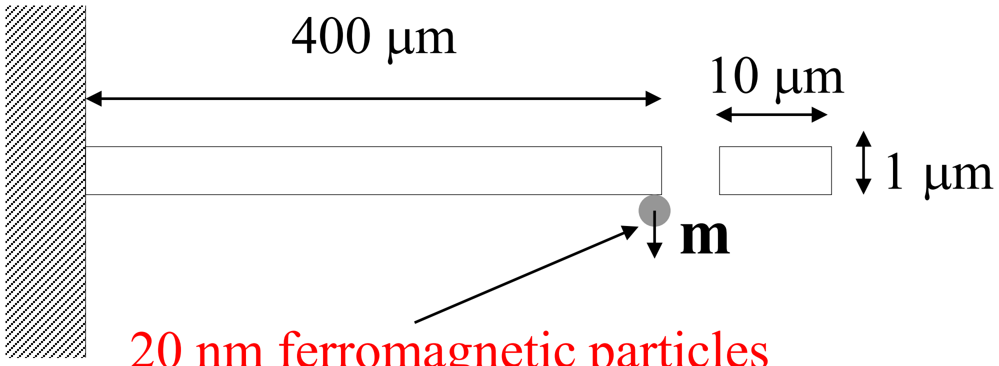
$$k \cong \frac{Ew}{4}\left(\frac{h}{l}\right)^3$$

E: Young modulus (N/m^2)

k: spring constant (N/m)

MFM: example 1

Si cantilever (E=190x10^9 N/m^2, ρ=2333 kg/m)

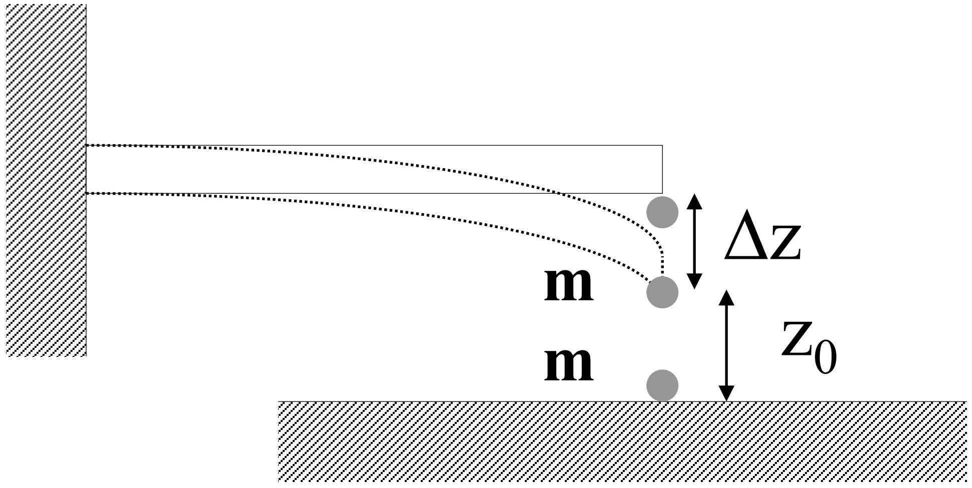


$Q \sim 1000$ (in air at 300 K)

$k \cong 0.01\,\mathrm{N/m}$

$\omega_c \cong 2\pi \times 9\mathrm{kHz}$

$|\mathbf{m}| \sim 3\mathrm{x}10^{-17}\ \mathrm{Am}^2$



$$\frac{\partial B}{\partial z}(0,0,z_0) \cong 2\times 10^4\,\mathrm{T/m}$$ (for z_0=100 nm)

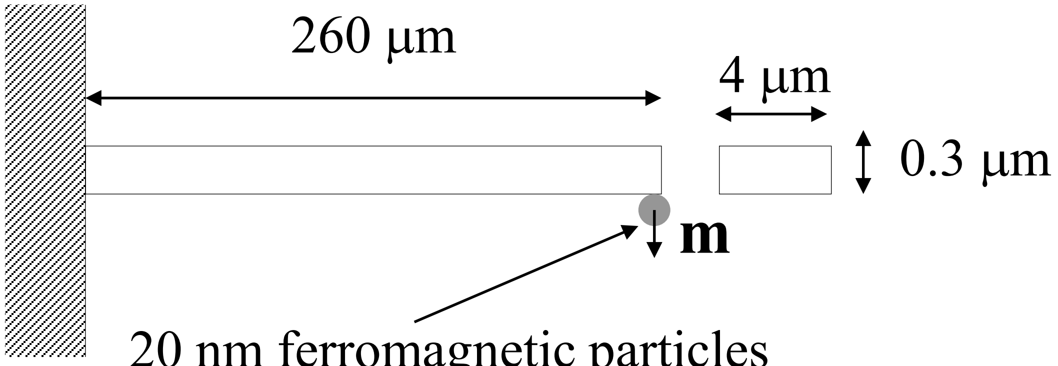
$$F_z = |\mathbf{m}|\frac{\partial B_z}{\partial z} \cong 5\times 10^{-14}\,\mathrm{N}$$

$$\Delta z = \frac{1}{k}F \cong 0.05\,\mathrm{Å}\ (\text{at } \omega = 0)$$

$$\Delta z = \frac{Q}{k}F \cong 5\,\mathrm{nm}\ (\text{at } \omega = \omega_c)$$

MFM: example 2

Si cantilever (E=190x10⁹ N/m², ρ=2333 kg/m)

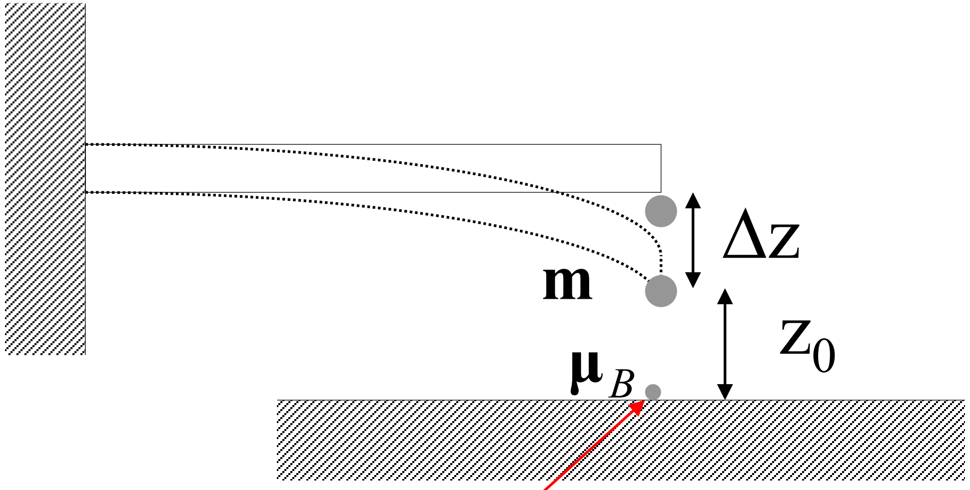


$Q \sim 150000$ (in vacuum at 0.3 K)

$k \cong 0.00026\,\mathrm{N/m}$

$\omega_c \cong 2\pi \times 6\,\mathrm{kHz}$

$|\mathbf{m}| \sim 3\times10^{-17}\ \mathrm{Am^2}$



$\boldsymbol{\mu}_B \cong 10^{-23}\,\mathrm{Am^2}$

$$\frac{\partial B}{\partial z}(0,0,z_0) \cong 1\,\mathrm{T/m}$$ (for z_0=50 nm)

$$F_z = |\mathbf{m}|\frac{\partial B_z}{\partial z} \cong 3\times10^{-18}\,\mathrm{N}$$

$$\Delta z = \frac{1}{k}F \cong 0.0001\,\text{Å}\ (\text{at } \omega = 0)$$

$$\Delta z = \frac{Q}{k}F \cong 2\,\mathrm{nm}\ (\text{at } \omega = \omega_c)$$

MFM: noise

The dominant source is, usually, the thermal motion of the cantilever:

$$\Delta z_{rms} \cong \sqrt{\frac{4k_B TQ}{k\omega_c}}\sqrt{\Delta f}$$

(at $\omega=\omega_c$)

$$\Delta z_{rms} \cong \sqrt{\frac{4k_B T}{Qk\omega_c}}\sqrt{\Delta f}$$

(at $\omega<<\omega_c$)

$$F_{rms} \cong \sqrt{\frac{4kk_B T}{Q\omega_c}}\sqrt{\Delta f}$$

$$\omega_c \cong \sqrt{\frac{E}{\rho}}\frac{h}{l^2}$$

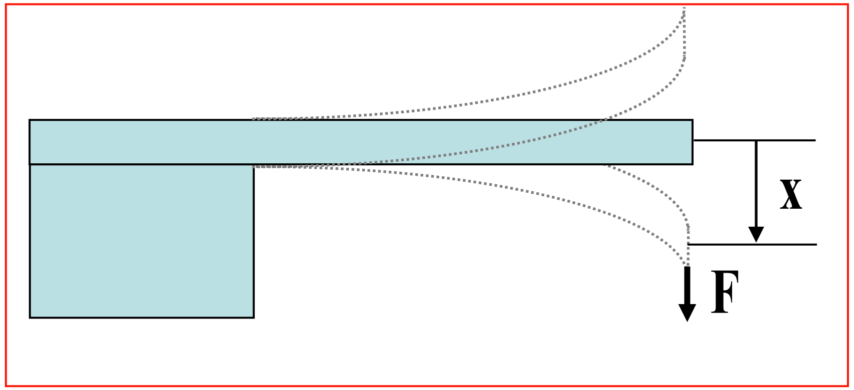
Q: Cantilever quality factor (unknown a priori)

ω_c: Cantilever resonance frequency (Hz)

ρ: Cantilever density (kg/m^3)

The minimum detectable force at DC or at resonance is the same

Detection of a **force** with a **cantilever**



$$x(\omega \ll \omega_0) = \frac{F}{k}, \quad x(\omega \cong \omega_0) = Q\frac{F}{k}$$

$$x_{NC} = \sqrt{\langle x^2 \rangle} = ???$$

$$SNR_{INT} = \frac{x}{x_{NC}} = ????$$

A powerfull tool:
The Fluctuation-Dissipation Theorem

Classical or quantum system in thermal equilibrium:

$$H_0 = H_0(Q_1, Q_2, ...)$$

H_0 : Hamiltonian

Q_i : Observable

Fluctuation-Dissipation Theorem

$$S_{Q_i}(\omega) = ...$$

$$\langle Q_i^2 \rangle = ...$$

Spectral density of the fluctuations of Q_i

RMS value of the fluctuations of Q_i

Fluctuation-Dissipation Theorem: Formalism

$$H = H_0 + FQ$$

$$R(\omega) \equiv Re\left\{\frac{F(\omega)}{\dot{Q}(\omega)}\right\}$$

$$G(\omega) \equiv Re\left\{\frac{\dot{Q}(\omega)}{F(\omega)}\right\}$$

$$\langle Q^2 \rangle = \frac{2}{\pi}\int_0^\infty E(\omega,T)\frac{1}{\omega^2}G(\omega)d\omega$$

$$\langle \dot{Q}^2 \rangle = \frac{2}{\pi}\int_0^\infty E(\omega,T)G(\omega)d\omega$$

$$\langle F^2 \rangle = \frac{2}{\pi}\int_0^\infty E(\omega,T)R(\omega)d\omega$$

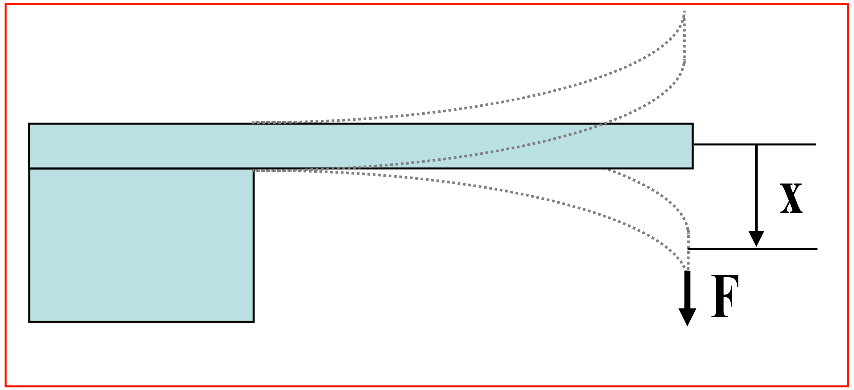
$$S_Q = \frac{2}{\pi}\frac{1}{\omega^2}E(\omega,T)G(\omega)$$

$$S_{\dot{Q}} = \frac{2}{\pi}E(\omega,T)G(\omega)$$

$$S_F = \frac{2}{\pi}E(\omega,T)R(\omega)$$

$$E(\omega,T) = \hbar\omega\left[\frac{1}{\exp(\hbar\omega/k_BT)-1}+\frac{1}{2}\right] \cong k_BT \quad (\text{for } k_BT \gg \hbar\omega)$$

Detection of a force with a cantilever

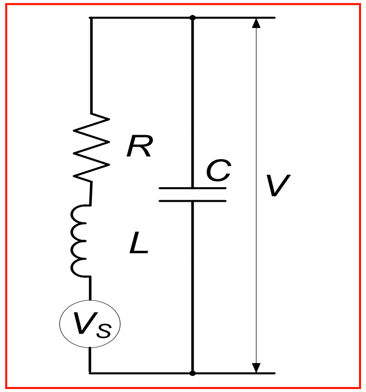


$$m\ddot{x}(t)+\lambda\dot{x}(t)+kx(t)=F(t)$$

$$\omega_0 \equiv \sqrt{\frac{k}{m}} \qquad Q \equiv \omega_0 m/\lambda$$

$$\left|\frac{x(\omega)}{F(\omega)}\right| = \frac{1}{k}\frac{1}{\sqrt{\left(1-(\omega/\omega_0)^2\right)^2+\left((\omega/\omega_0)(1/Q)\right)^2}}$$

Detection of a magnetic field with a coil



$$|V_S(\omega)| \cong \omega A_{ff}\,|B(\omega)|$$

$$L\ddot{V}(t)+R\dot{V}(t)+(1/C)V(t)=V_S(t)$$

$$\omega_0 \equiv \sqrt{\frac{1}{LC}} \qquad Q \equiv \omega_0 L/R$$

$$\left|\frac{V(\omega)}{B(\omega)}\right| = \omega A_{ff}\sqrt{\frac{1}{\left((\omega/\omega_0)^2-1\right)^2+\left((\omega/\omega_0)(1/Q)\right)^2}}$$

Applications of the Fluctuation-Dissipation Theorem

Cantilever

$$m\ddot{x}(t)+\lambda\dot{x}(t)+kx(t)=F(t)$$

$$H_0=\frac{1}{2}m\dot{x}^2+\frac{1}{2}kx^2$$

$$H=H_0+F(t)x(t)$$

$$S_x(\omega)\cong\frac{2k_BT}{\pi\omega_0 Q}\frac{1}{k}\frac{1}{\left(1-(\omega/\omega_0)^2\right)^2+\left((\omega/\omega_0)(1/Q)\right)^2}$$

$$S_x(\omega\cong\omega_0)\cong\frac{2k_BT}{\pi\omega_0}\frac{Q}{k}$$

$$\langle x^2\rangle=\int_0^\infty S_x(\omega)d\omega=\frac{k_BT}{k}$$

Coil

$$L\ddot{V}(t)+R\dot{V}(t)+(1/C)V(t)=V_S(t)$$

$$H_0=\frac{1}{2}LI^2+\frac{1}{2}CV^2$$

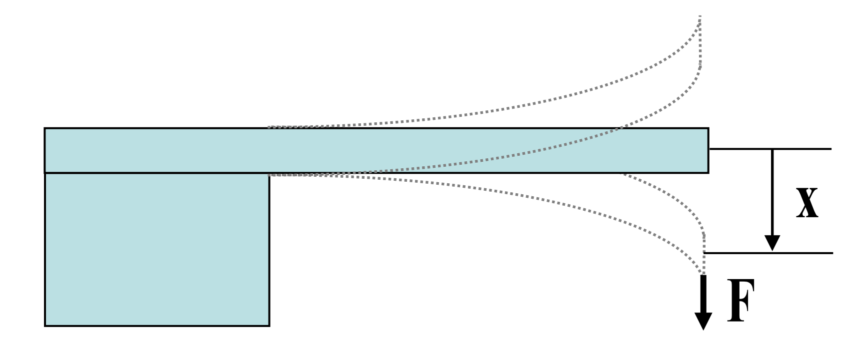
$$H=H_0+V_S(t)CV(t)$$

$$S_V(\omega)\cong\frac{2k_BT}{\pi\omega_0 Q}\frac{1}{C}\frac{1}{\left(1-(\omega/\omega_0)^2\right)^2+\left((\omega/\omega_0)(1/Q)\right)^2}$$

$$S_V(\omega\cong\omega_0)\cong\frac{2k_BT}{\pi\omega_0}\frac{Q}{C}=\frac{2k_BT}{\pi}RQ^2$$

$$\langle V^2\rangle=\int_0^\infty S_V(\omega)d\omega=\frac{k_BT}{C}$$

Cantilever



$$S_x(\omega \cong \omega_0) \cong \frac{2k_B T}{\pi}\frac{Q}{\omega_0 k}$$

$$S_F = \frac{2k_B T}{\pi}\frac{k}{\omega_0 Q}$$

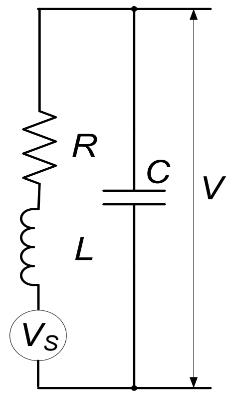
Magnetic field force
Light pressure
Electrostatic force
….

« Observable »

« Driving Force »

« Driving Physical Interaction »

Coil



$$|V_S(\omega)| \cong \omega A_{ff}|B(\omega)|$$

$$S_V(\omega \cong \omega_0) \cong \frac{2k_B T}{\pi}\frac{Q}{\omega_0 C}$$

$$S_{V_S} = \frac{2k_B T}{\pi}\frac{1}{\omega_0 QC}$$

AC magnetic field

$$S_B = \left(\frac{1}{\omega_0 A_{eff}}\right)^2 \frac{2k_B T}{\pi}\frac{1}{\omega_0 QC}$$

Standard silicon cantilever at 300 K

E=190x10^9 N/m^2, ρ=2333 kg/m^3

l=400 μm, w=10 μm, h=1 μm

k=0.01 N/m, $\omega_0/2\pi$=9 kHz

T=300 K, Q=10000

$$\sqrt{S_x(\omega \ll \omega_0)} = 2\times10^{-14} \ \mathrm{m}/\sqrt{\mathrm{Hz}}$$

$$\sqrt{S_x(\omega \cong \omega_0)} = 2\times10^{-10} \ \mathrm{m}/\sqrt{\mathrm{Hz}}$$

$$\sqrt{S_F(\omega \ll \omega_0)} = 2\times10^{-16} \ \mathrm{N}/\sqrt{\mathrm{Hz}}$$

$$\sqrt{S_F(\omega \cong \omega_0)} = 2\times10^{-16} \ \mathrm{N}/\sqrt{\mathrm{Hz}}$$

Thin silicon cantilever at 0.3 K

E=190x10^9 N/m^2, ρ=2333 kg/m^3

l=260 μm, w=4 μm, h=0.3 μm

k=2.6x10^{-4} N/m, $\omega_0/2\pi$=4.98 kHz

T=0.3 K, Q=150000

$$\frac{S_x(\omega_0)_{SQL}}{S_x(\omega_0)} \cong 10^{-6}$$

$$\sqrt{S_x(\omega \ll \omega_0)} = 7\times10^{-16} \ \mathrm{m}/\sqrt{\mathrm{Hz}}$$

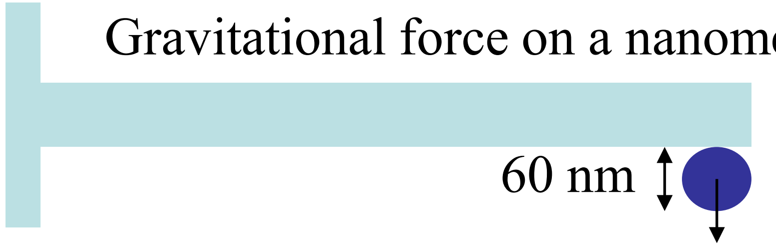
$$\sqrt{S_x(\omega \cong \omega_0)} = 1\times10^{-10} \ \mathrm{m}/\sqrt{\mathrm{Hz}}$$

$$\sqrt{S_F(\omega \ll \omega_0)} = 1\times10^{-18} \ \mathrm{N}/\sqrt{\mathrm{Hz}}$$

$$\sqrt{S_F(\omega \cong \omega_0)} = 1\times10^{-18} \ \mathrm{N}/\sqrt{\mathrm{Hz}}$$

Best force sensitivity reported !

Gravitational force on a nanometric water drop



m=10^{-19} kg

F=10^{-18} N

MFM: amplitude, frequency, phase detection

$$\Delta z \cong \frac{Q}{k} F$$

Amplitude detection

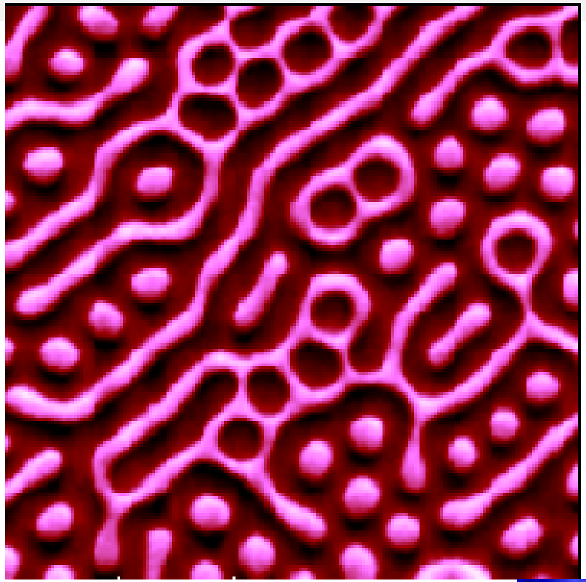
$$\Delta\omega \cong \frac{1}{2k}\frac{\partial F}{\partial z}$$

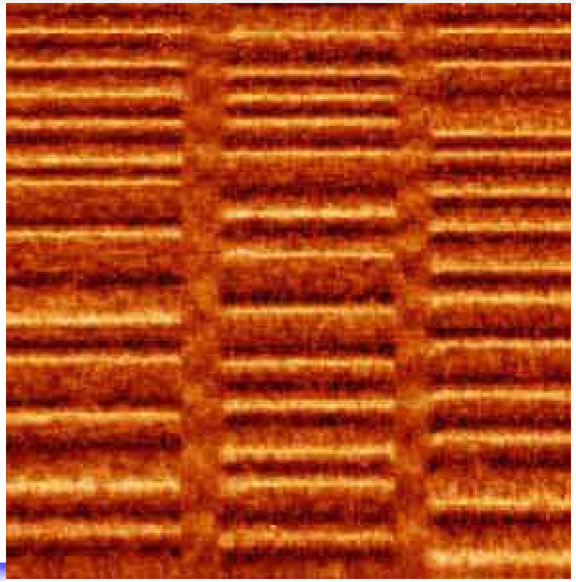
Frequency detection

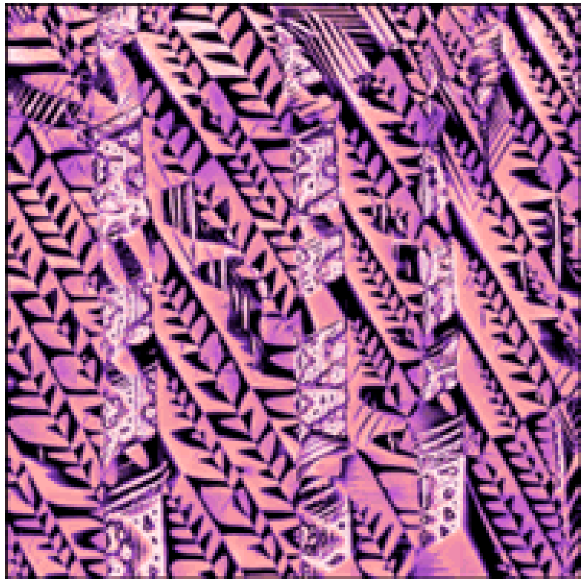
$$\alpha \cong \mathrm{arctg}\,\frac{\omega_c \omega}{2Q}\frac{1}{\omega^2 - \omega_c}$$

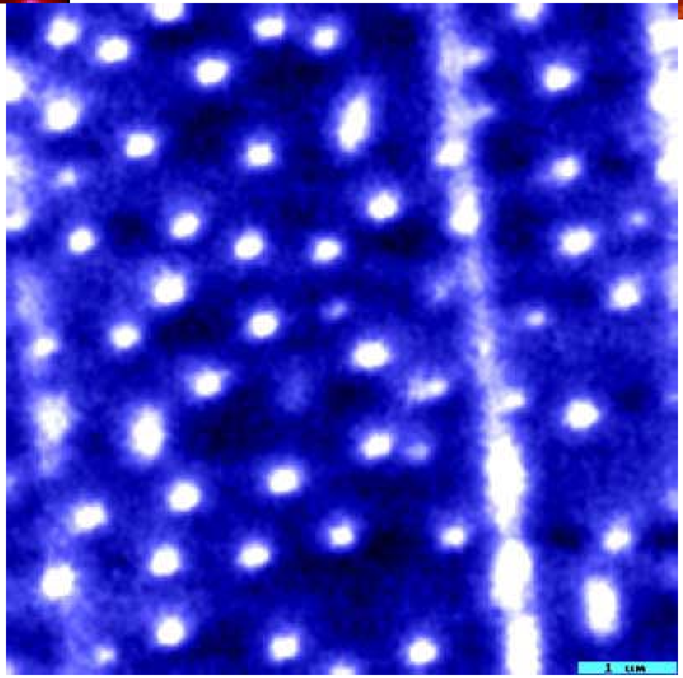
Phase detection

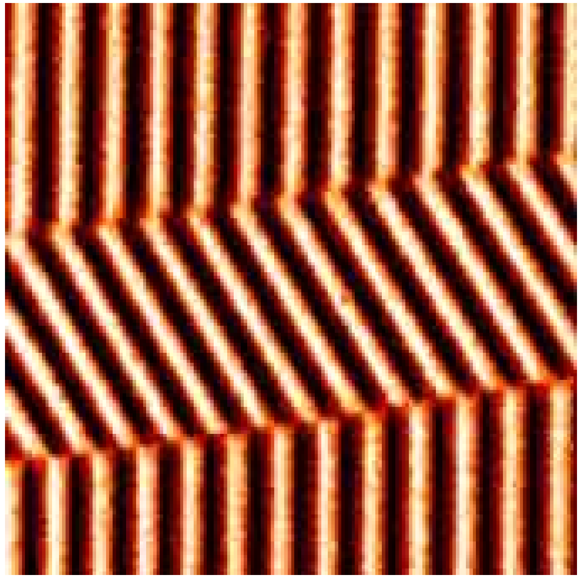
MFM: image gallery



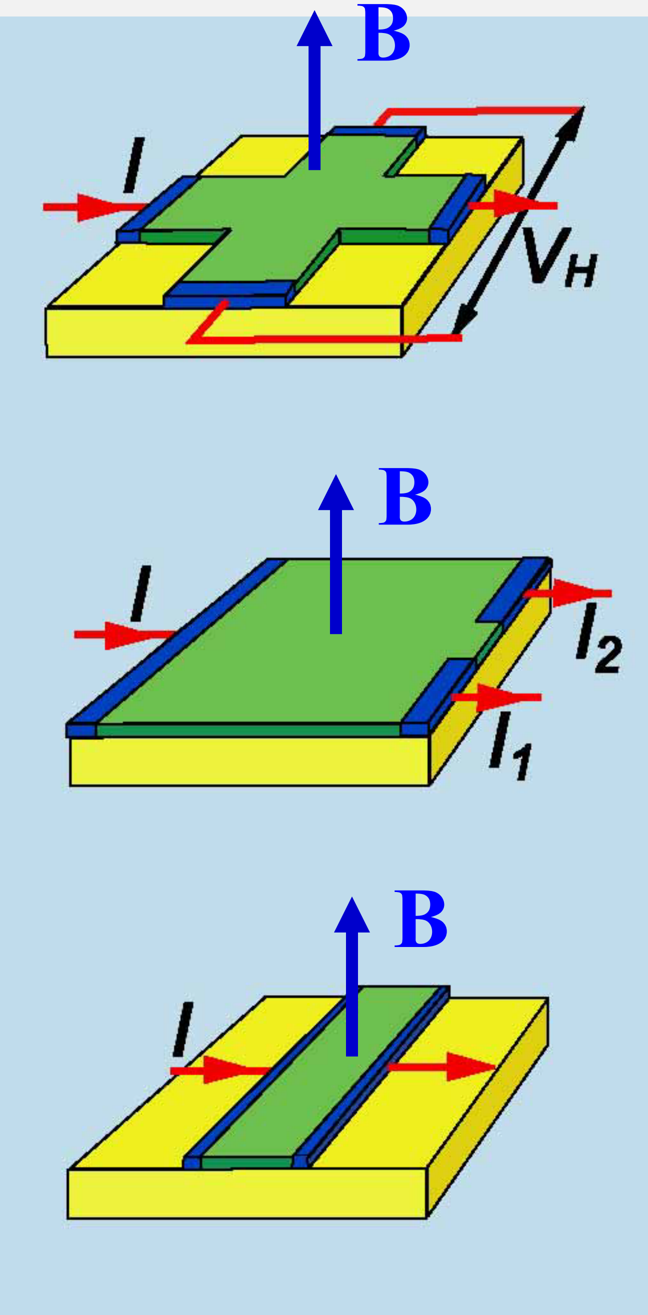








Scanning Hall probe microscopy (SHPM)



Hall cross

$$V_H = S_I \, I \, B$$

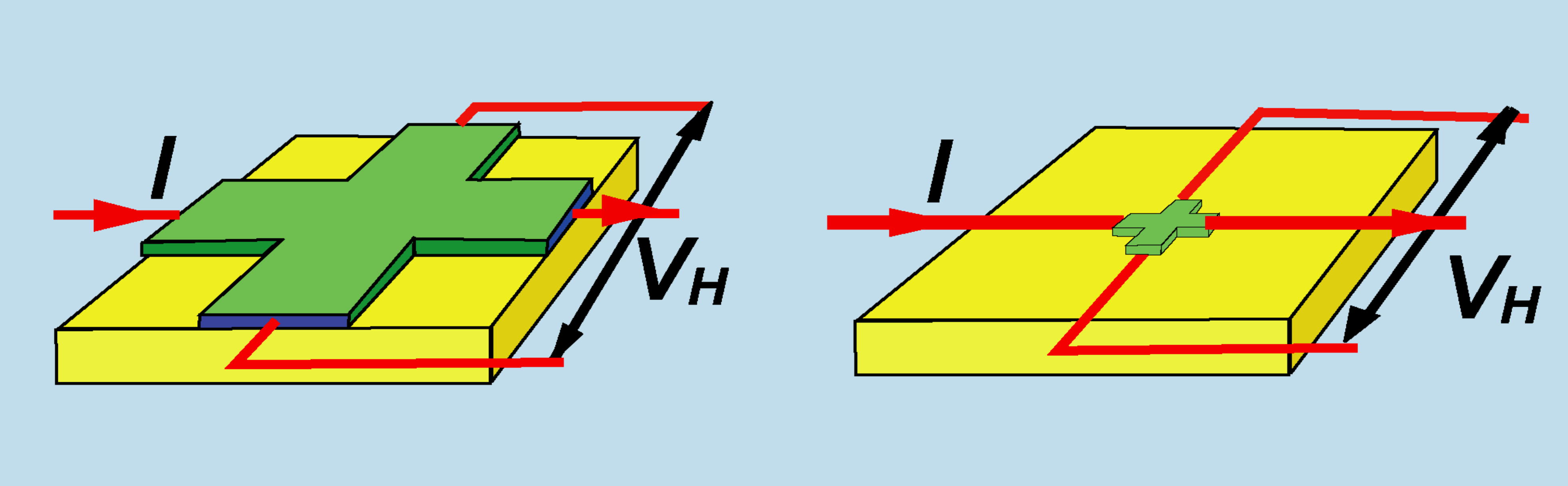
Split current

$$I_1 - I_2 = S_{Ic} \, I \, B$$

Magnetoresistance

$$R \cong R_0 (1 + \mu^2 B^2)$$

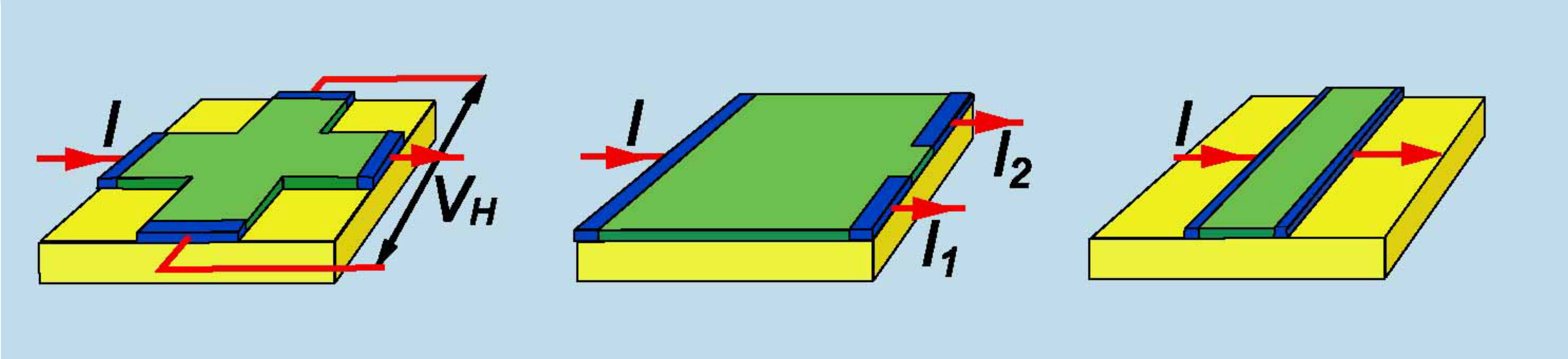
SHPM: scaling-down



Scaling-down main problems:

- Heating (same power in a smaller volume)
- Saturation of the drift velocity (10^5 m/s @ 1 V/μm)
- Larger 1/f noise (smaller amount of carriers)

SHPM: magnetic field resolution



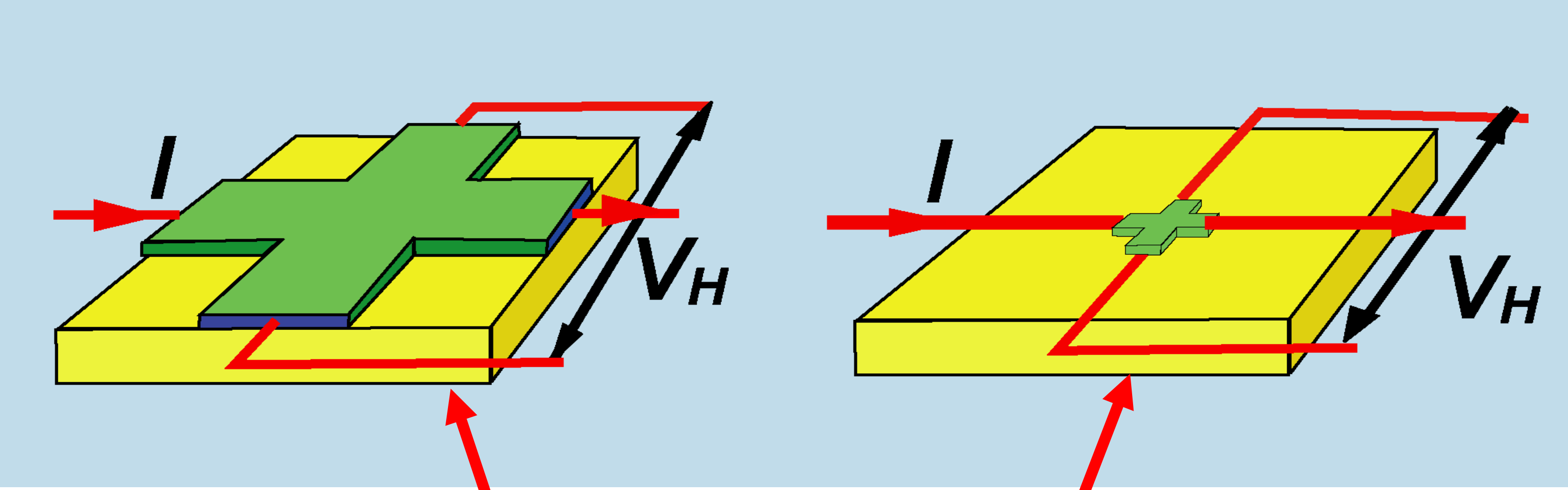
$$B_{\min} \equiv \frac{\sqrt{4\,k\,T\,R}}{V_H / B}$$

$$B_{\min} \equiv \frac{\sqrt{4\,k\,T\,/\,R}}{\Delta I / B}$$

$$B_{\min} \equiv \frac{\sqrt{4\,k\,T\,R}}{I(\partial R / \partial B)}$$

$$B_{\min} \cong \frac{\sqrt{4\,k\,T}}{v_{sat}\sqrt{\mu e n}\sqrt{wlt}} \cong \frac{\sqrt{4\,k\,TR_0}}{v_{sat} w} \qquad \left(\frac{T}{\sqrt{Hz}}\right)$$

SHPM: key features



	Macro	Micro	Unit
Dimensions	10 ÷ 1000	0.1 ÷ 10	μm
Field resolution	0.01 ÷ 100	1 ÷ 100000	nT/√Hz
Bandwidth	0 ÷ 1000		MHz
Resistance	0.01 ÷ 100		kΩ
Operating temp.	0 ÷ 500		K
Operating field	0 ÷ 30		T

SHPM: examples

at 300 K:

Type	Area (μm^2)	S_I (V/AT)	B_{min} ($nT/\sqrt{Hz}$)	@ I (mA)
Bi	$(0.1)^2$	3.3	70000	0.04
GaAs	$(0.3)^2$	30	130	2
2DEG	$(0.4)^2$	230	180	0.3
2DEG	$(0.8)^2$	300	4000	0.003
2DEG	$(1.5)^2$	700	300	0.1
2DEG	$(2.0)^2$	350	5	2
Si	$(2.4)^2$	87	60	2
GaAs	$(4.0)^2$	3100	10	0.3
InSb	$(4.5)^2$	140	4	0.3

SHPM: technologies

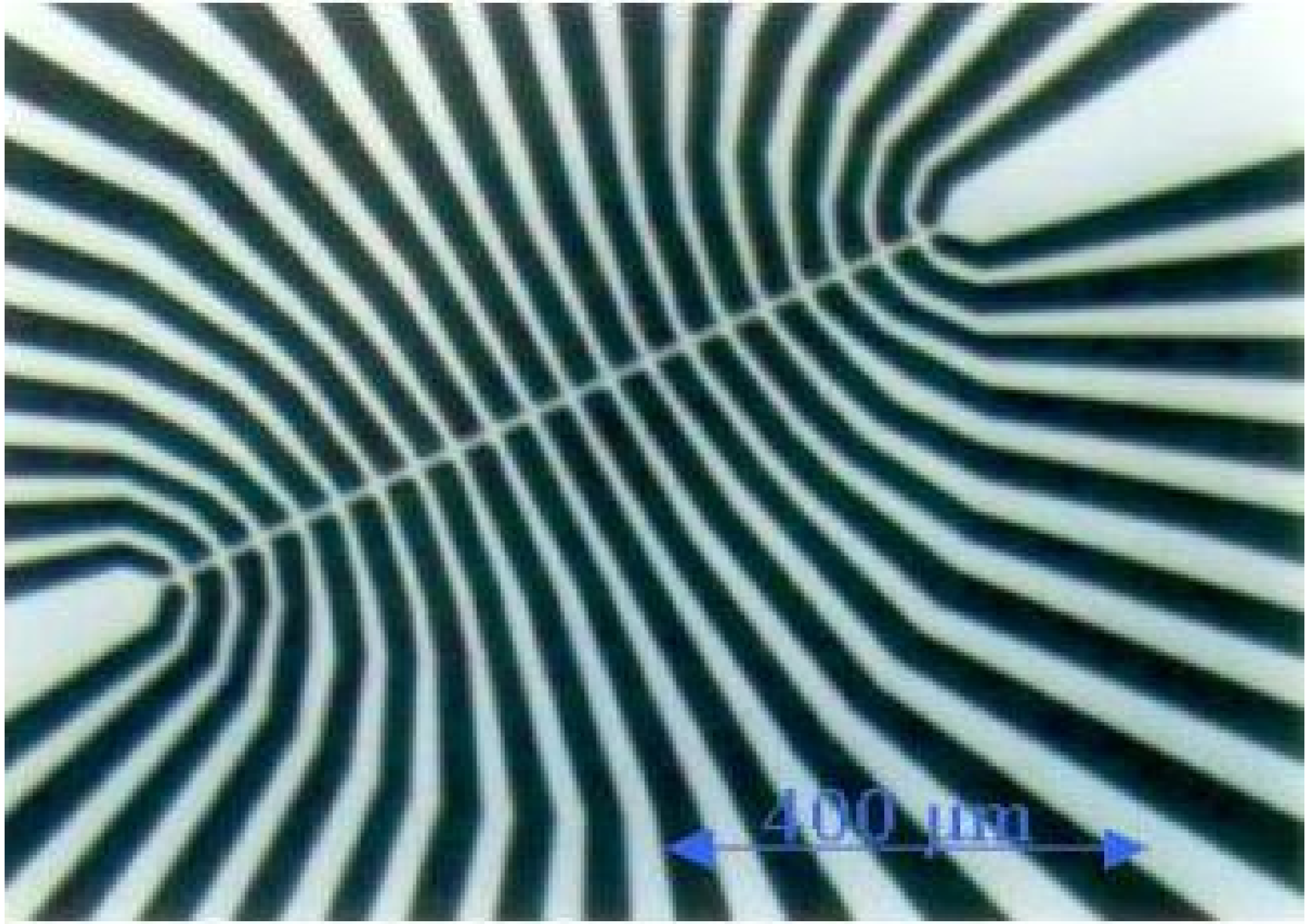
- *Thin films growth:* MOCVD
  MBE
  Evaporation
  Electrodeposition
  Maskless ion implantation

- *Thin films structuring:* Photolithography
  e-beam lithography
  Focused ion beam lithography

SHPM: applications

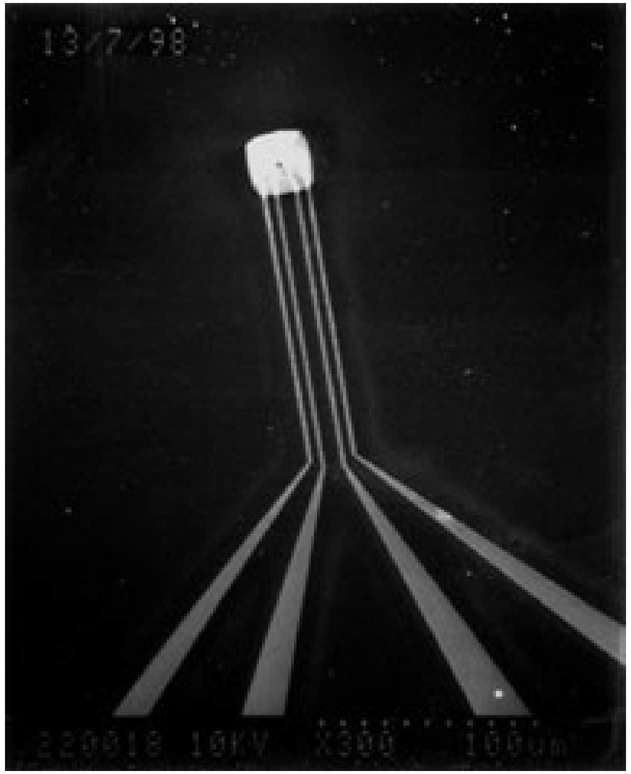
- Magnetic nano-particles
- Vortex in supercondutors
- Read heads for MRAMs
- Magnetic domains studies
- Microbeads for bio-chem. applications
- ESR spectroscopy (& imaging)
- …..

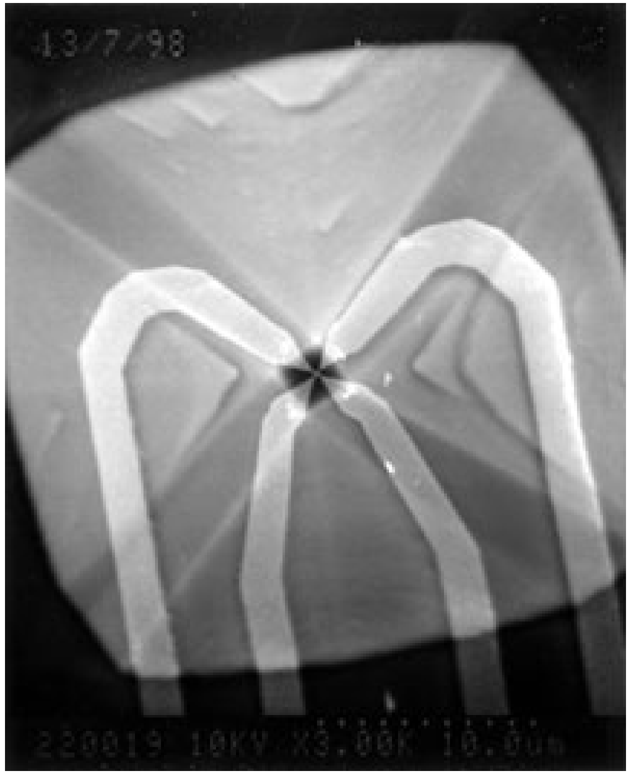
SHPM: devices



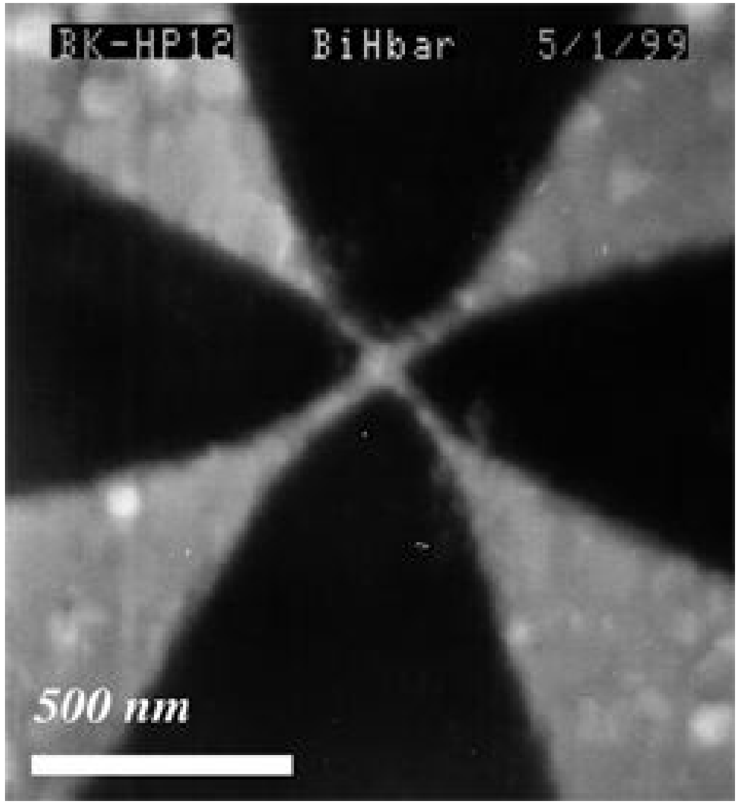
A one-dimensional array of 20 probes

Bi Hall sensor on a cantilever tip

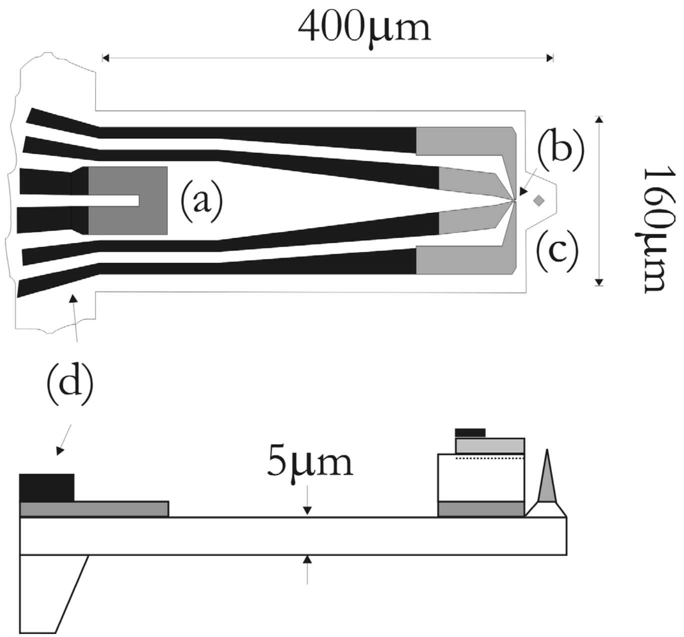


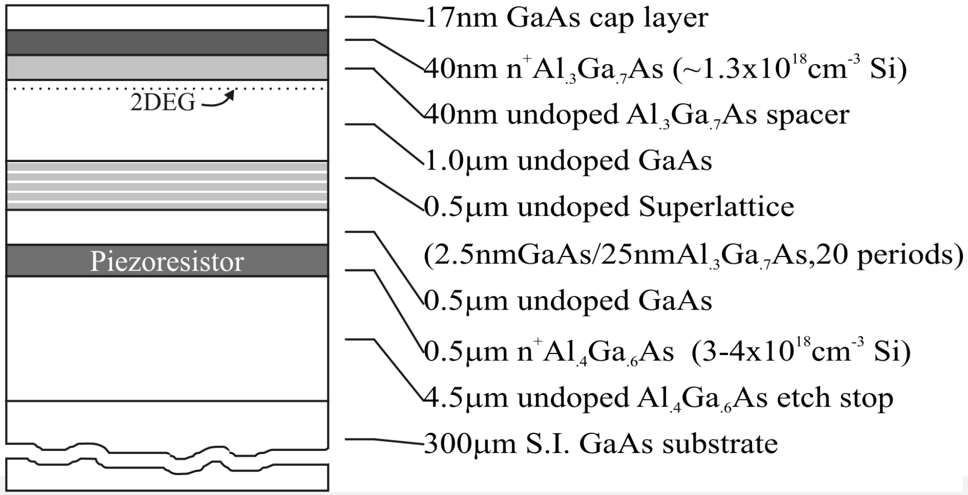


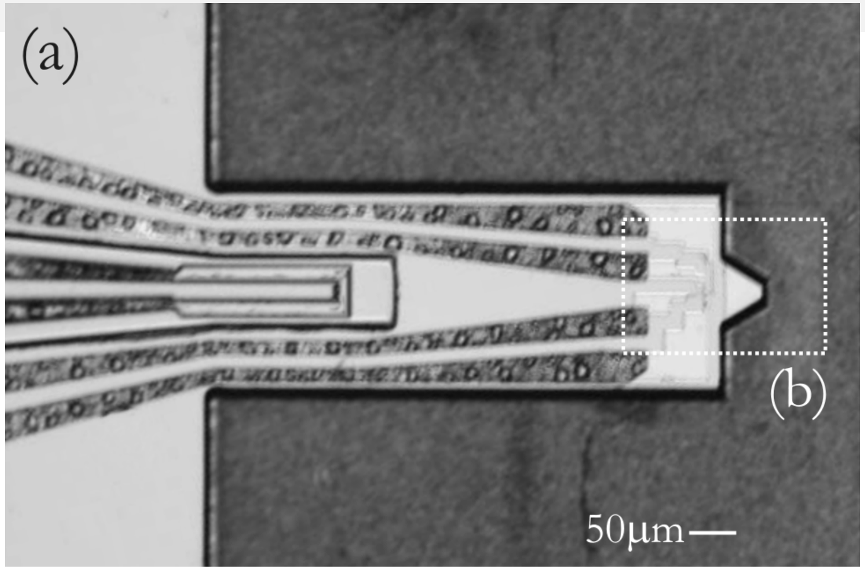
Bi on Si

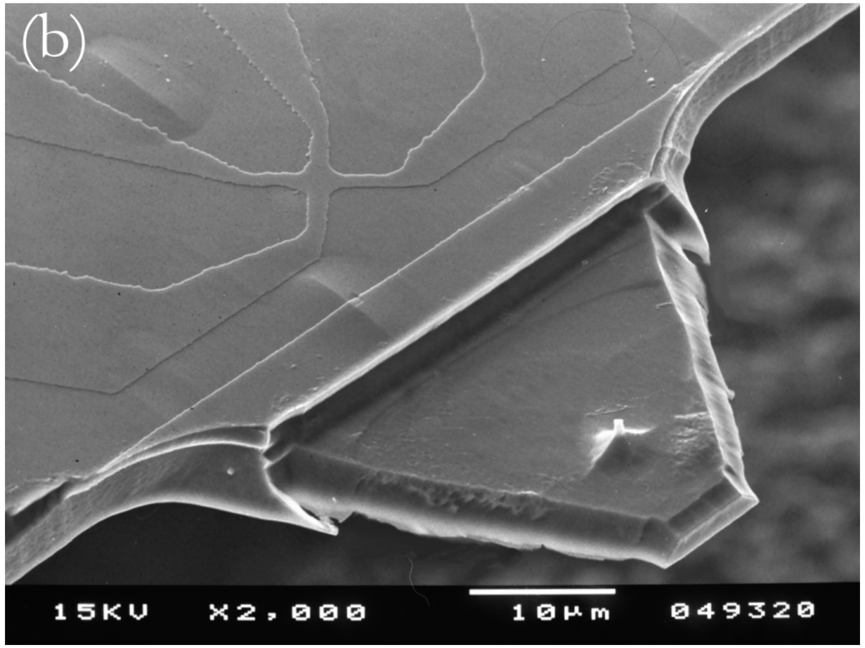


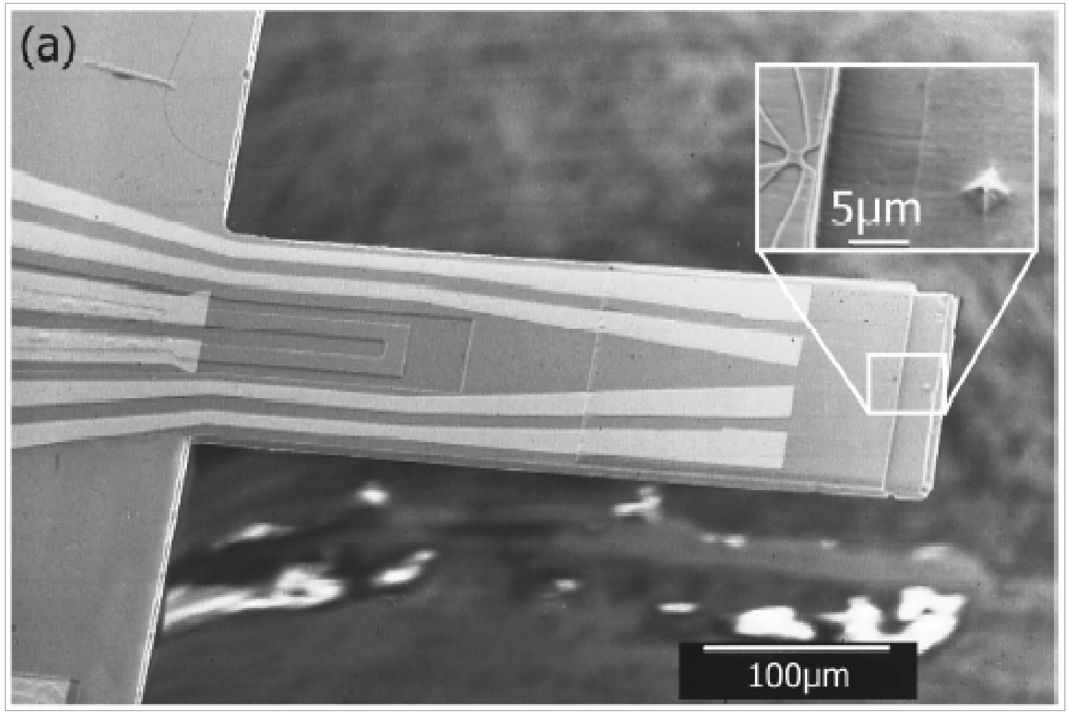
2DEG Hall sensor on a cantilever tip (with piezoresistor)

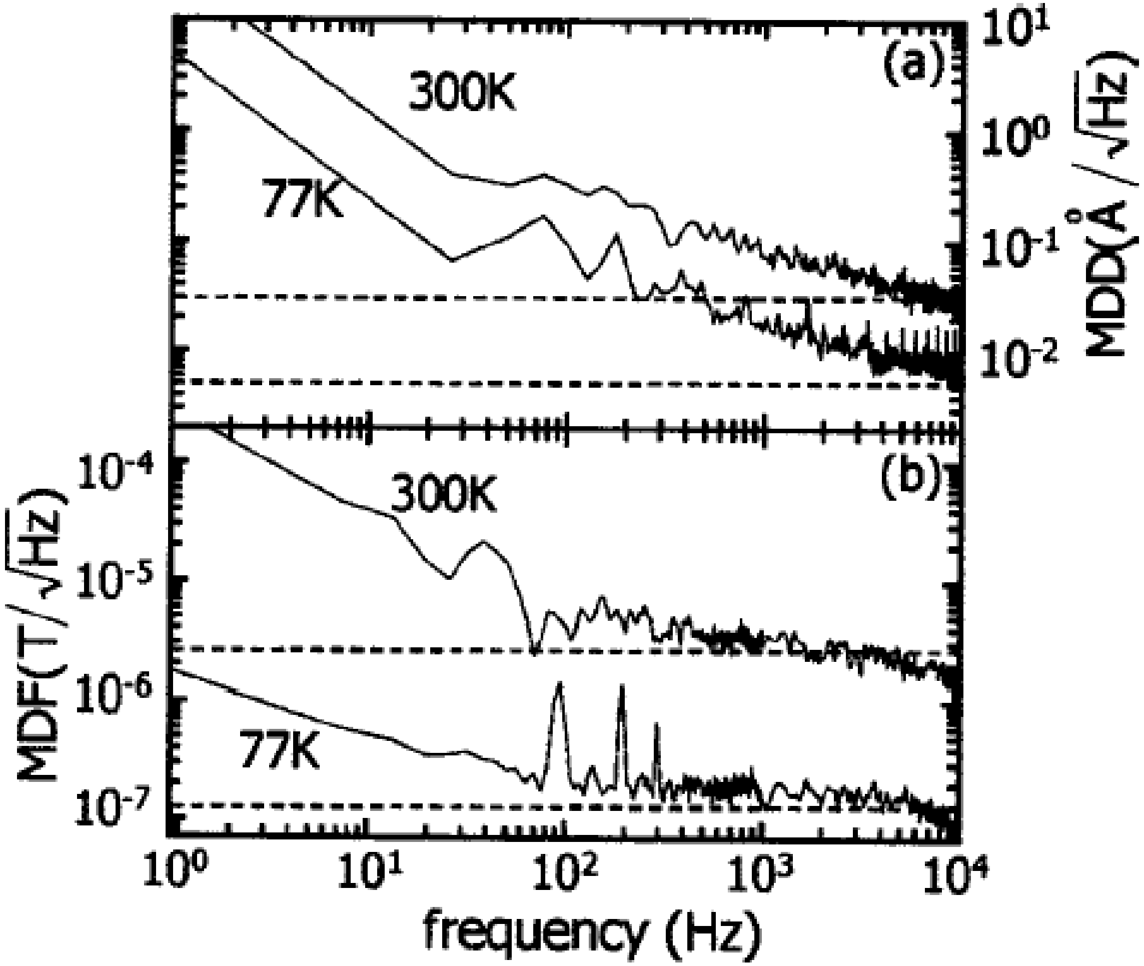




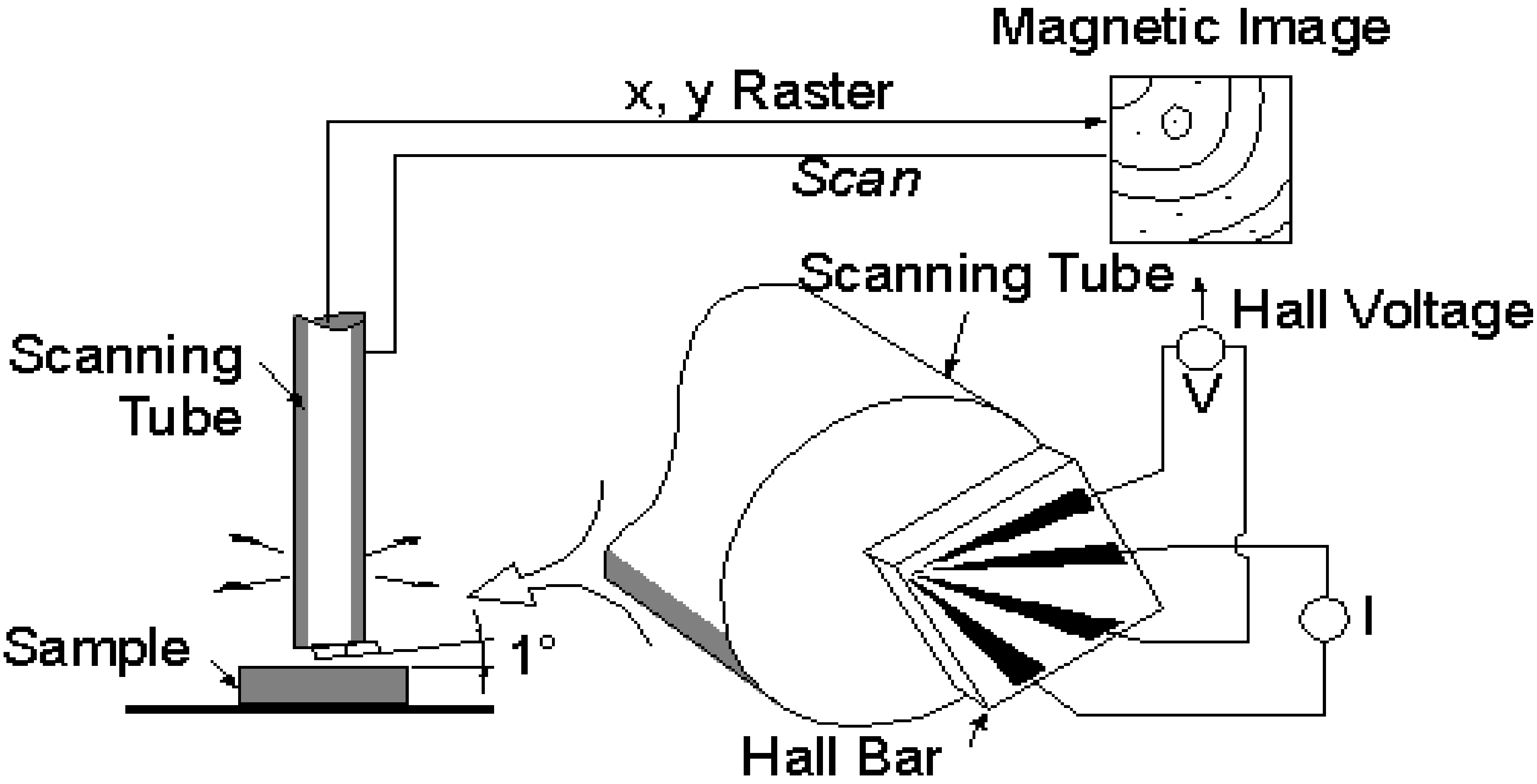





(a)
5µm
100µm

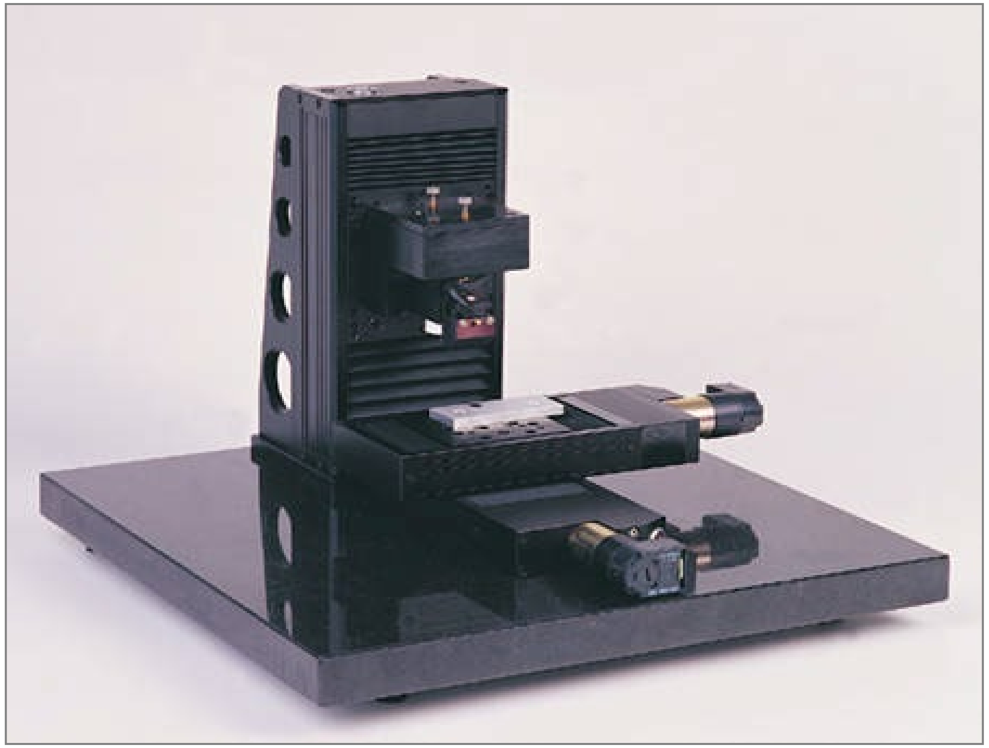

(a)
300K
77K
10^1
10^0
10^{-1}
10^{-2}
MDD(Å/√Hz)
(b)
300K
77K
10^{-4}
10^{-5}
10^{-6}
10^{-7}
MDF(T/√Hz)
10^0
10^1
10^2
10^3
10^4
frequency (Hz)

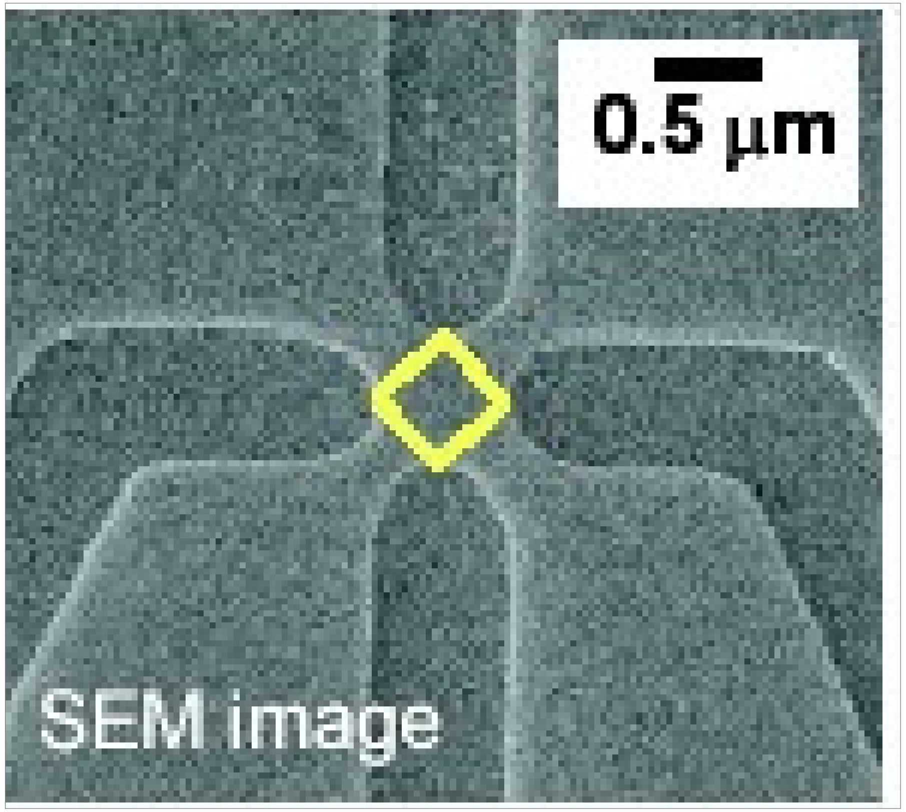
SHPM: complete set-up



SHPM: commercial system

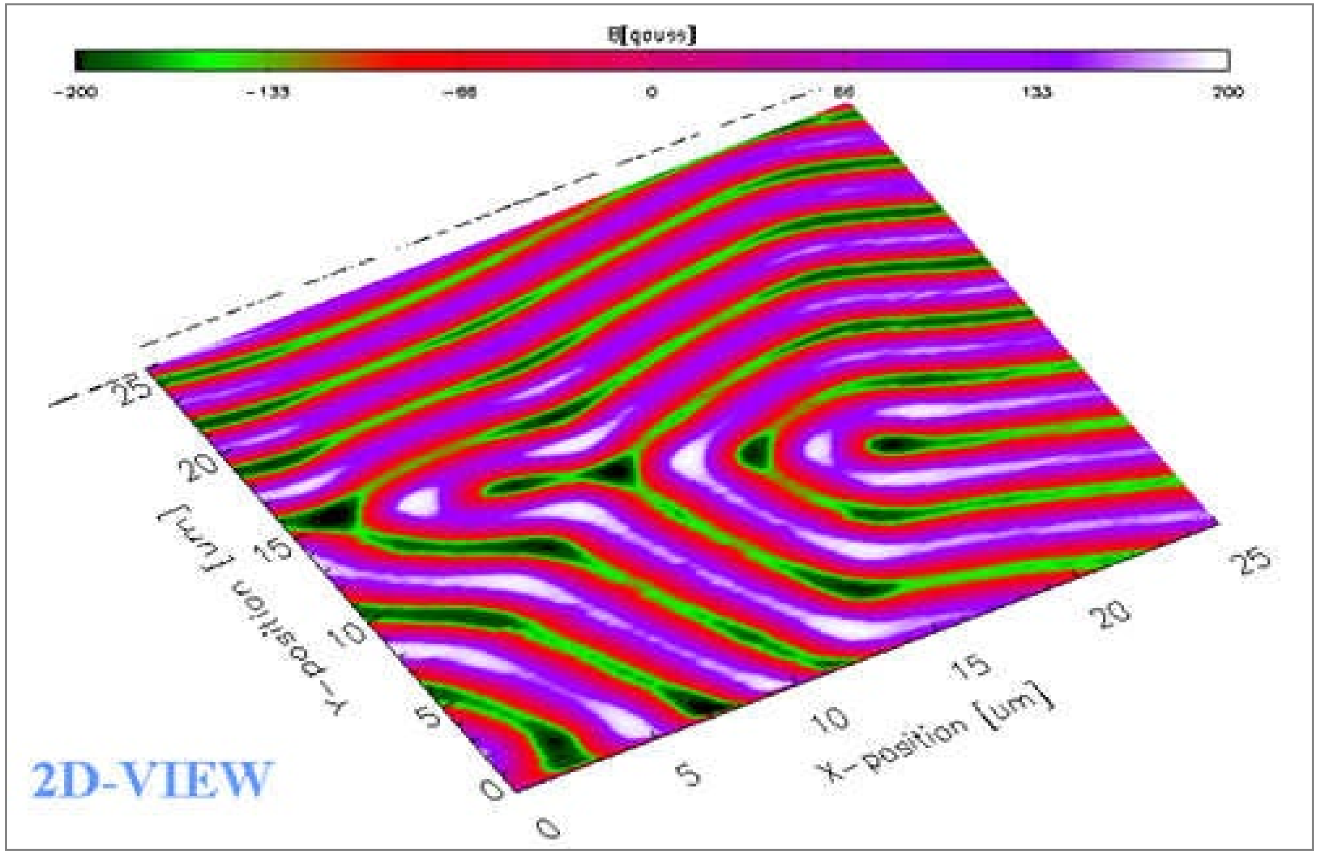
Scanning system

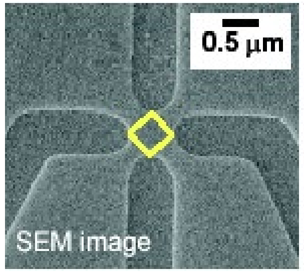


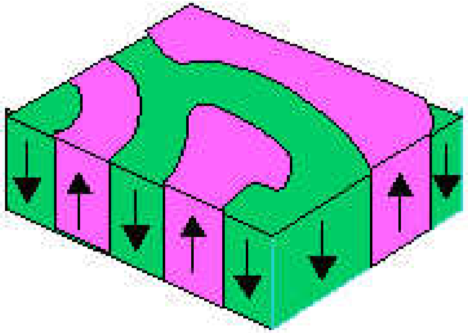


0.5 μm Hall cross

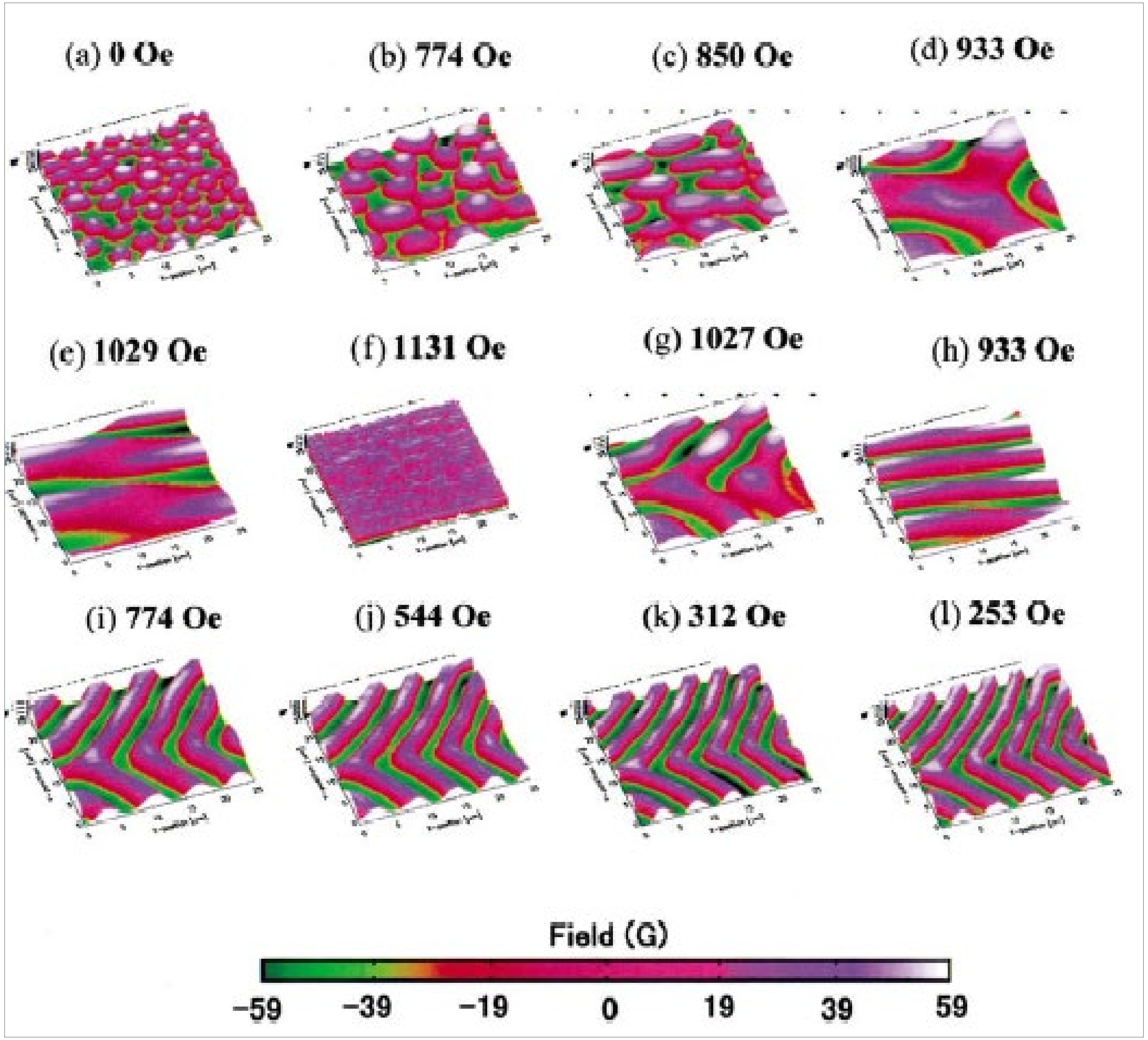
SHPM: image gallery



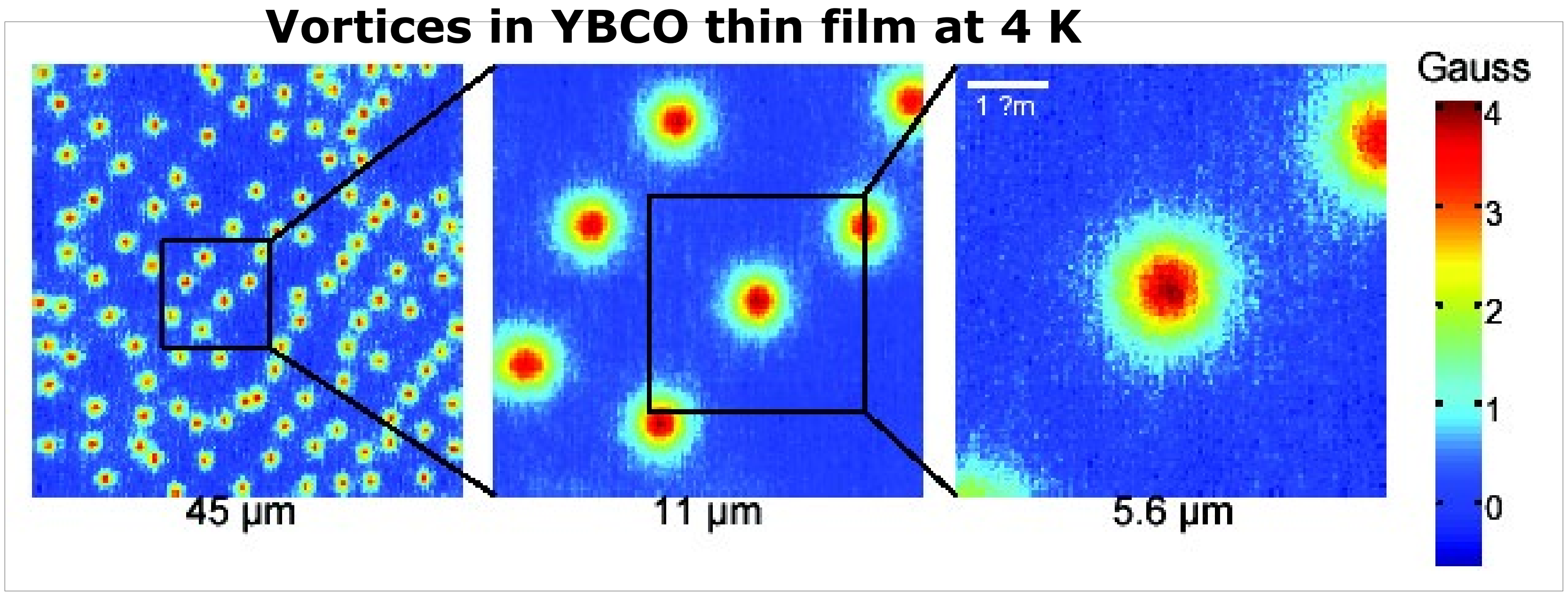


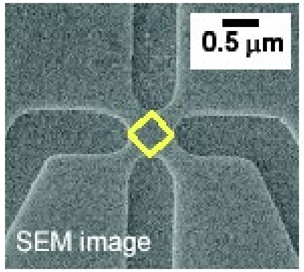


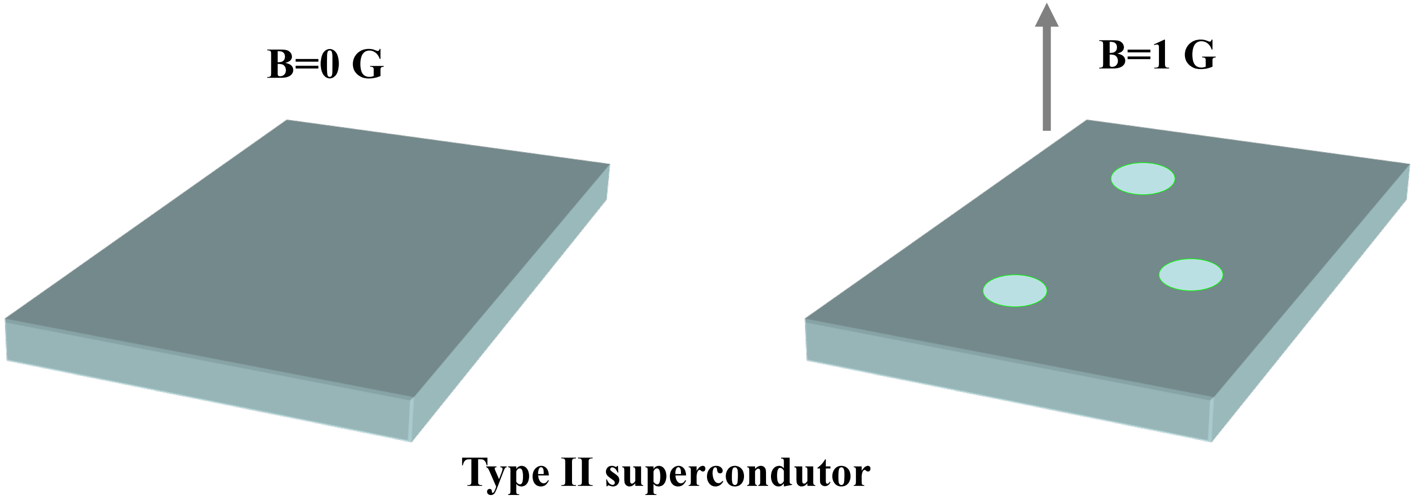
Bi-subst. iron garnet at 300K



(a) 0 Oe
(b) 774 Oe
(c) 850 Oe
(d) 933 Oe
(e) 1029 Oe
(f) 1131 Oe
(g) 1027 Oe
(h) 933 Oe
(i) 774 Oe
(j) 544 Oe
(k) 312 Oe
(l) 253 Oe
Field (G)
−59
−39
−19
0
19
39
59





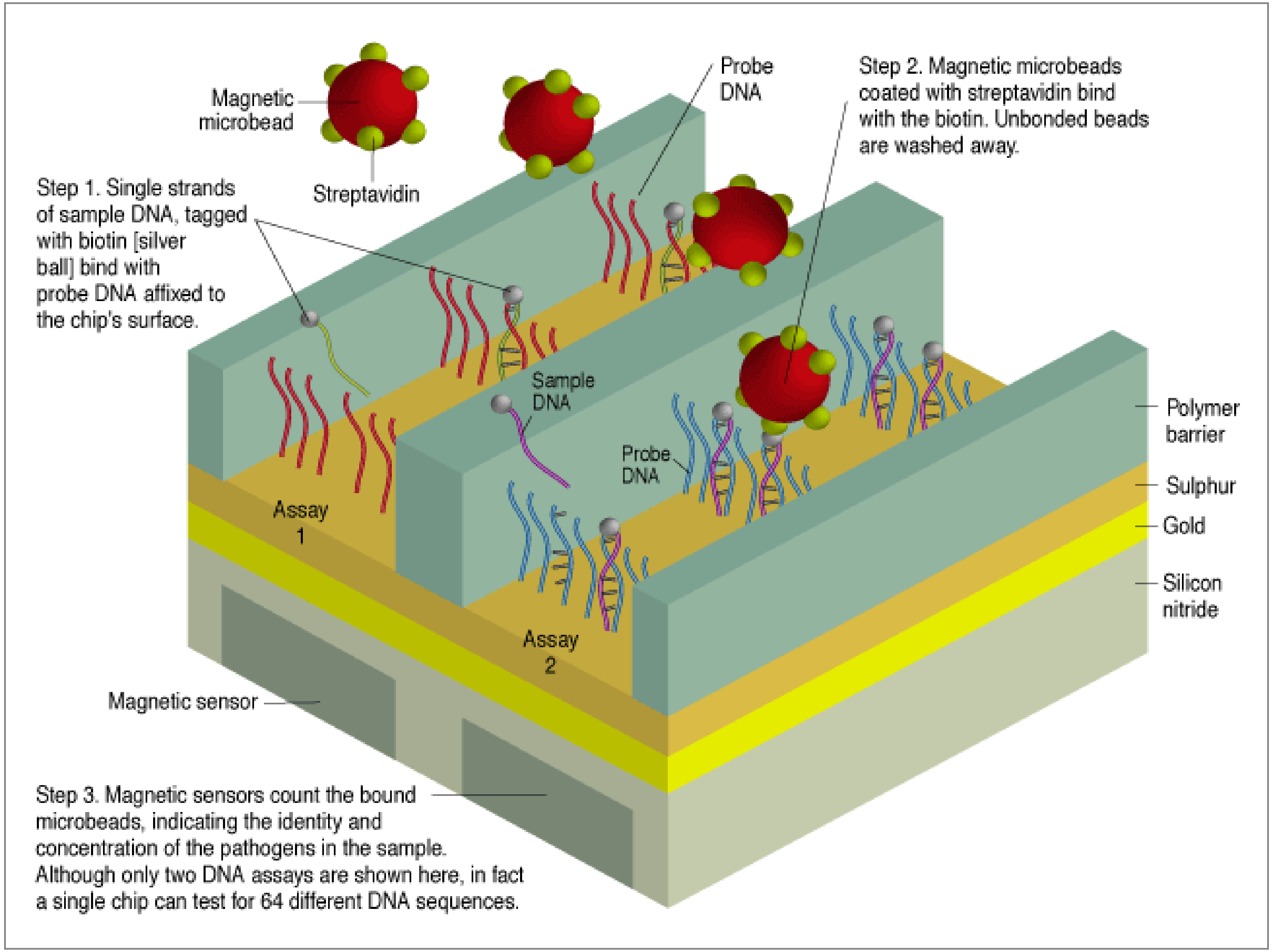


$$\Phi_B = n\Phi_0$$

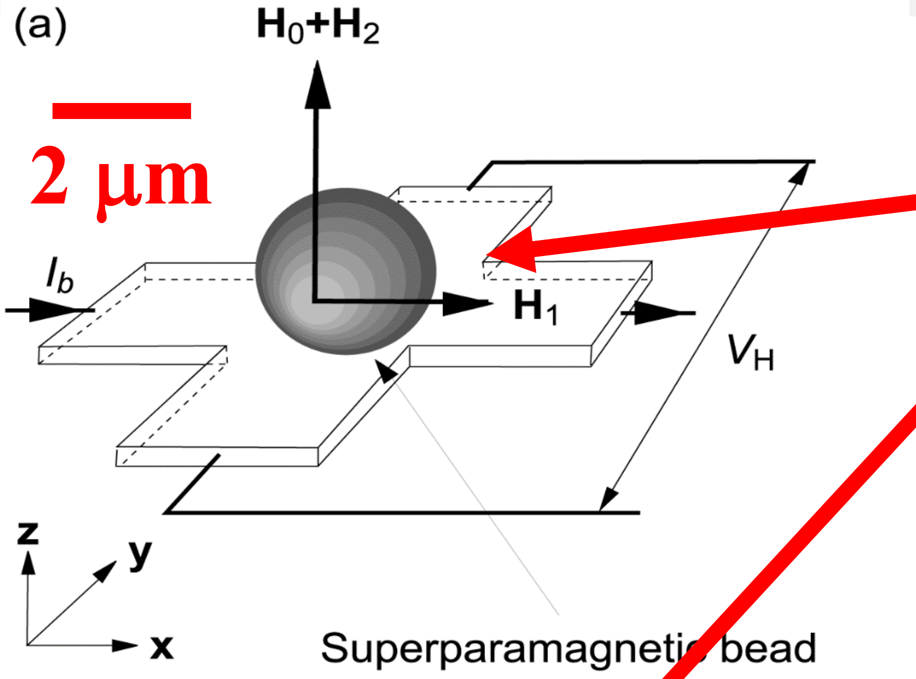
$$\Phi_0 \cong 2\times10^{-15}\,\mathrm{Tm}^2$$

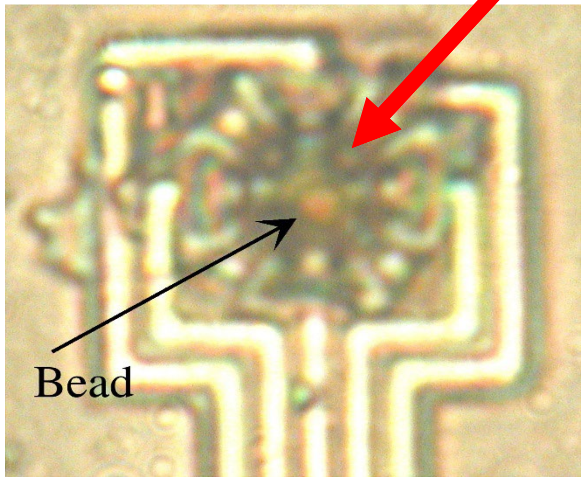
$$\frac{2\times10^{-15}\,\mathrm{Tm}^2}{(2\ \mu\mathrm{m})^2} \cong 5\ \mathrm{G}$$

SHPM: future bio-chemical applications



SHPM: future bio-chemical applications

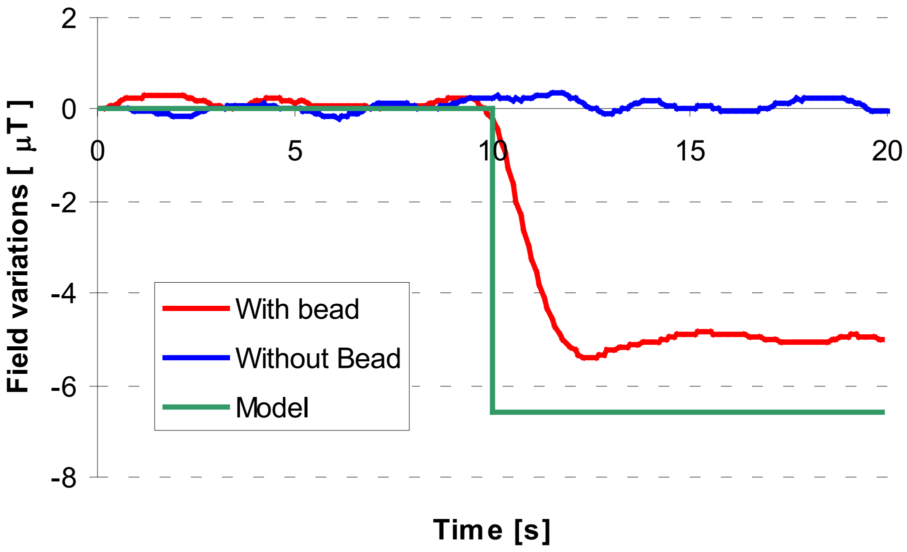




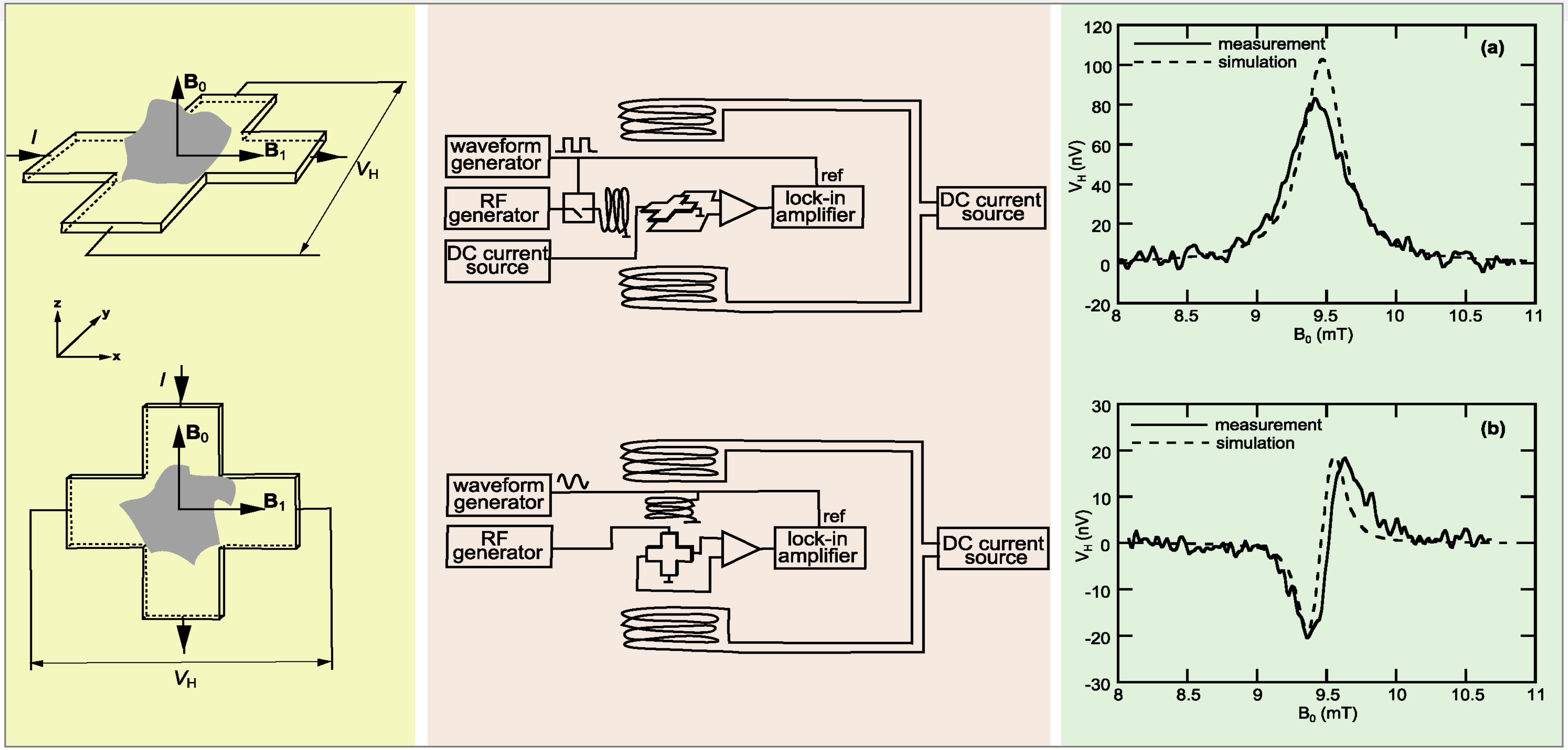
Hall sensor

Experiment:

- Beads: 2.8 µm Dynal Biotech
- Hall sensor: 2.4 µm Si CMOS
- T=300 K, P=1 atm
- DC and AC fields: 1 to 10 mT



P. A. Besse et al., *Appl. Phys. Lett.* **80**, 4199 (2001)

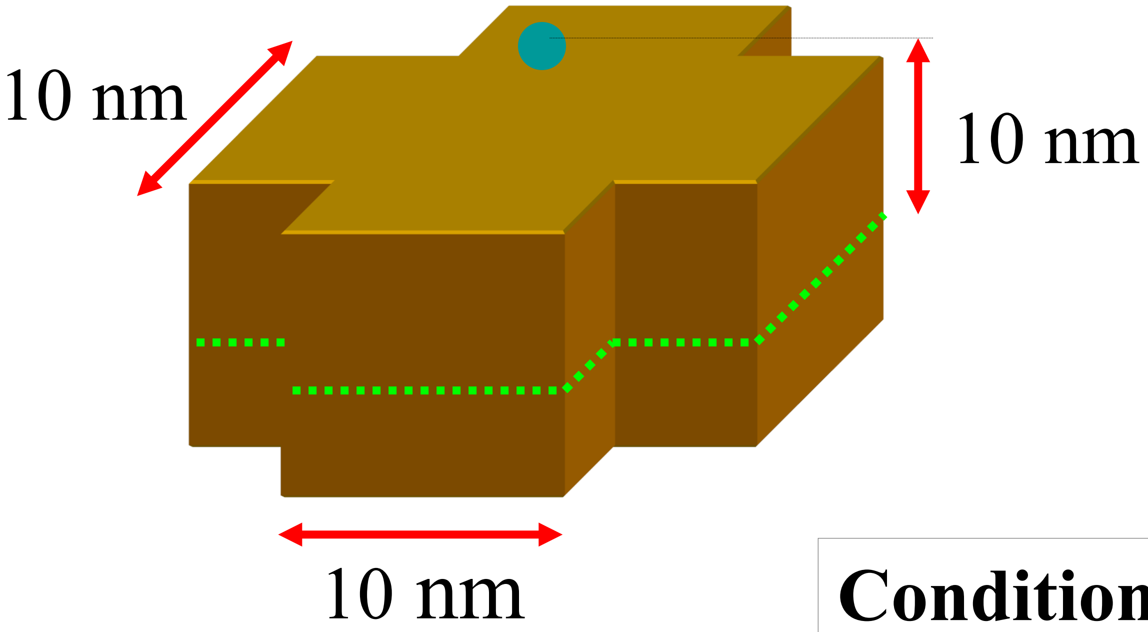


Electron spin resonance (ESR)

Experiment:
Hall Sensor: InSb, $(50\ \mu m)^2$
Sample: DPPH, $(50\ \mu m)^3$
Cond.: B_0=10 mT,T=300 K

G. Boero, P. A. Besse, R. S. Popovic, *Appl. Phys. Lett.* **79**, 1498 (2001)

SHPM: single electron spin detection ?



InGaAs/AlGaAs on GaAs (2DEG)

Conditions:

$I_{bias} = 1\ \mu A$

$S_I = 1000$ V/AT

$R = 10\ k\Omega$

$\Delta f = 1$ Hz

$T = 1$ K

Results:

$\langle B \rangle = 1\ \mu T$

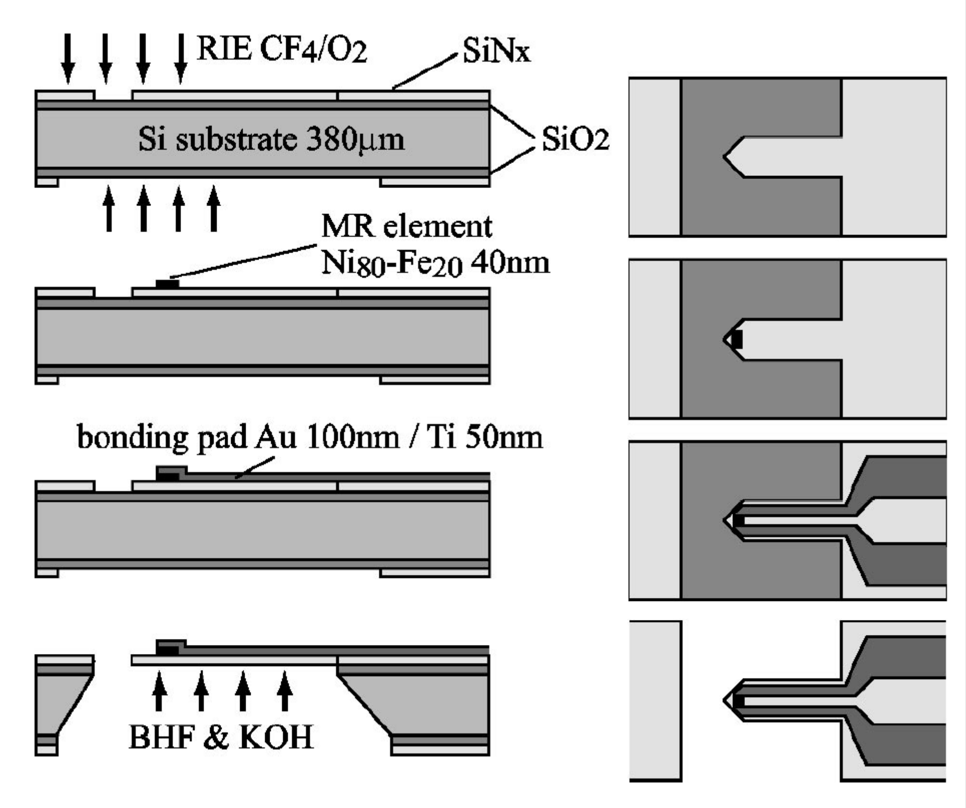
$V_H = 1$ nV

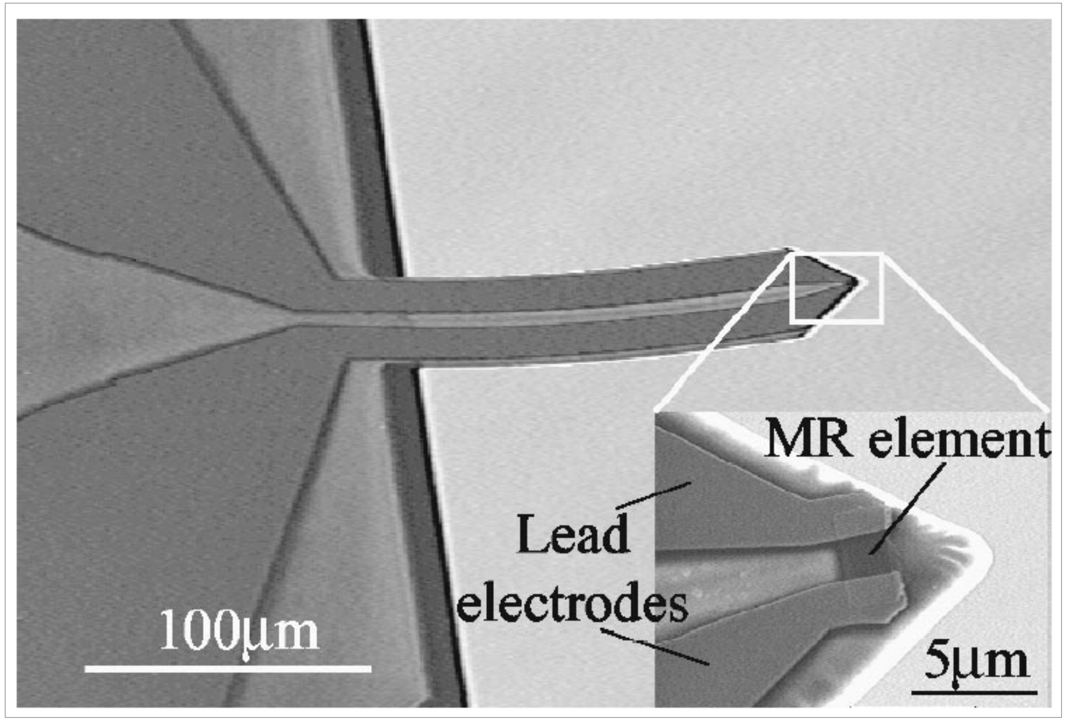
$N_{rms} = 1$ nV

$S/N = 1$

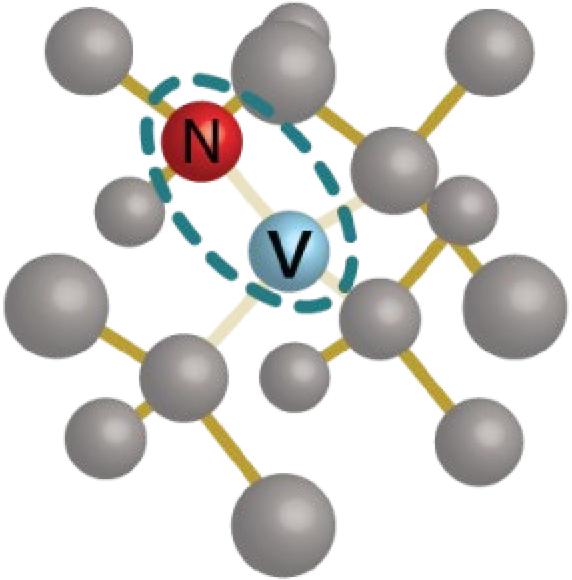
SHPM: similar techniques

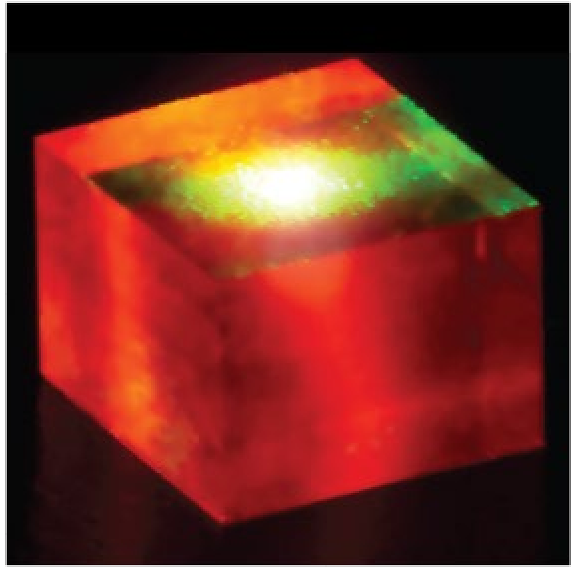
Scanning magnetoresistance

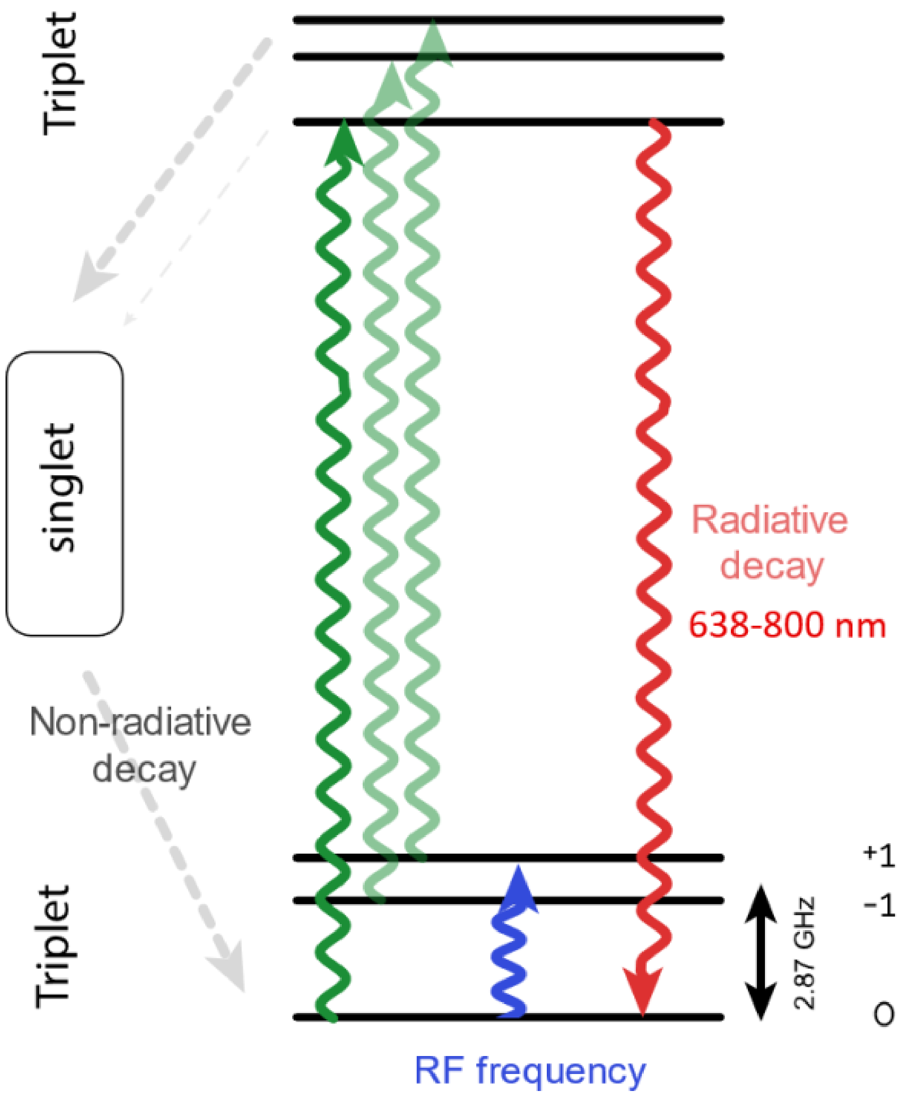




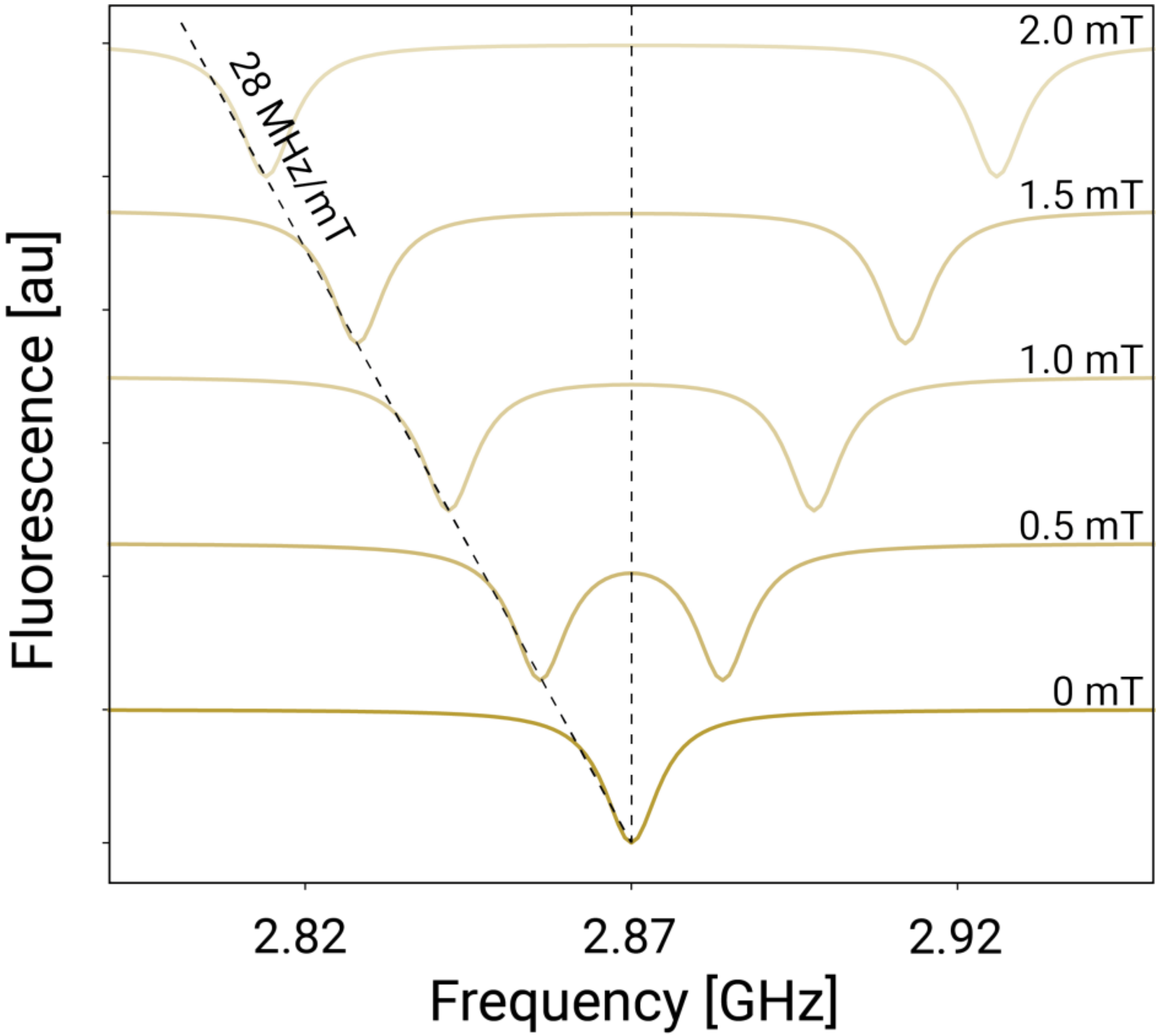
NV centers in diamond based magnetic imaging



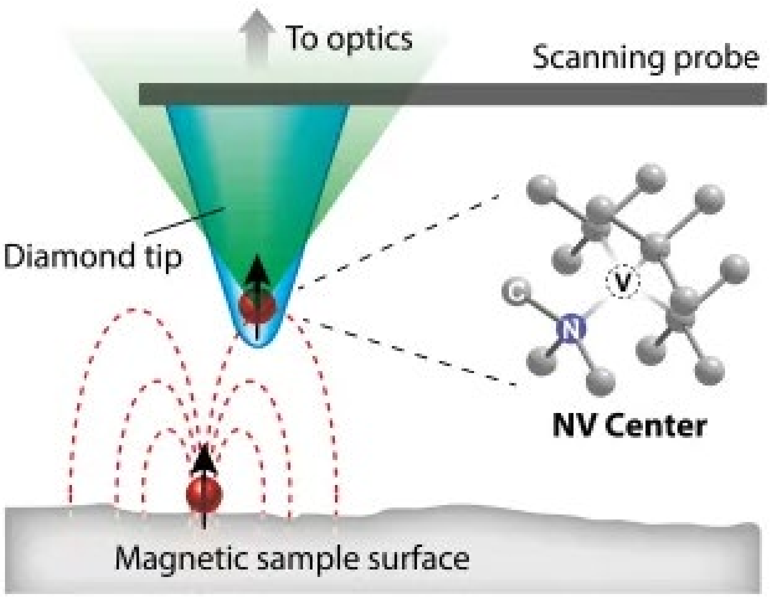


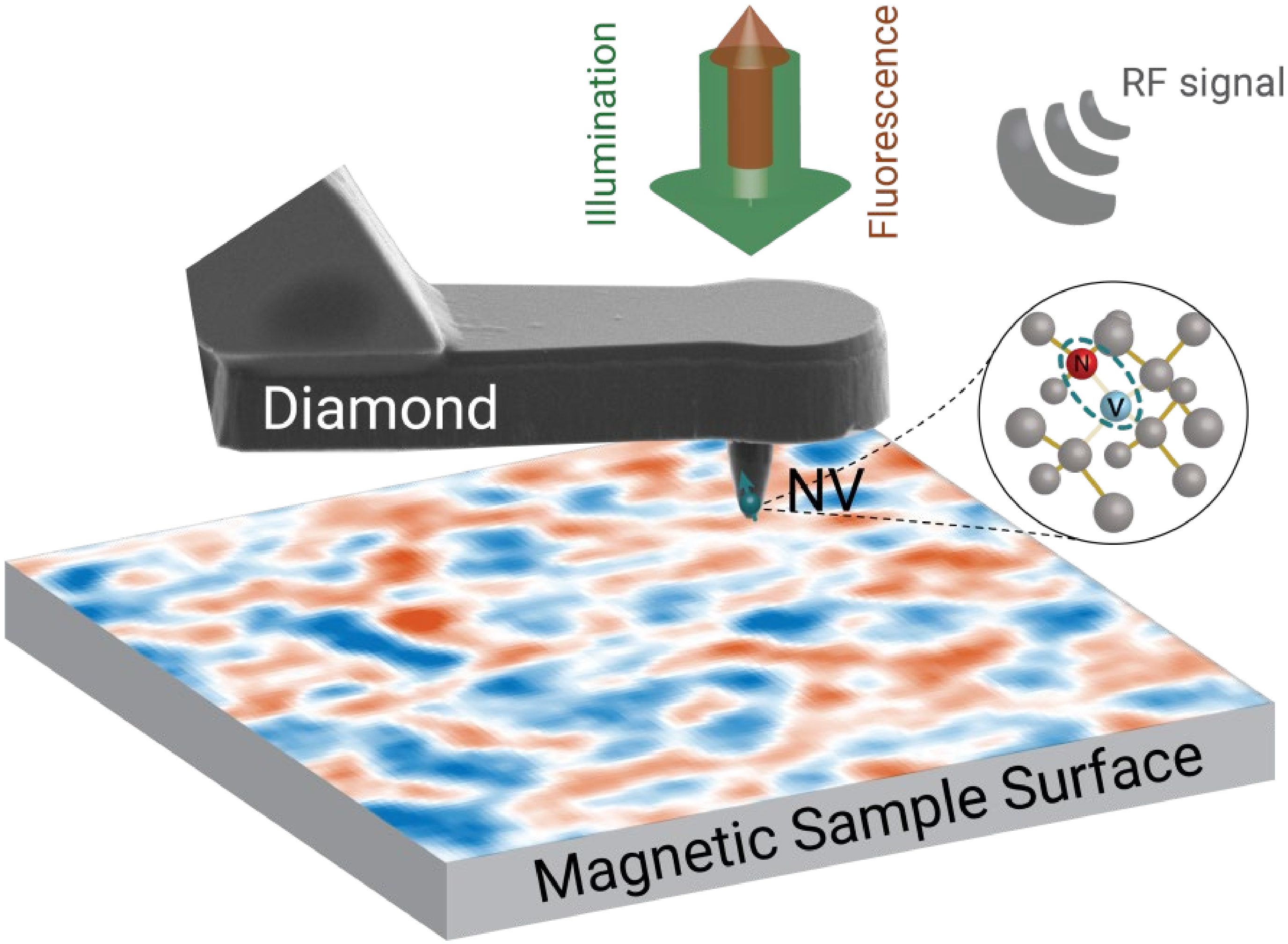


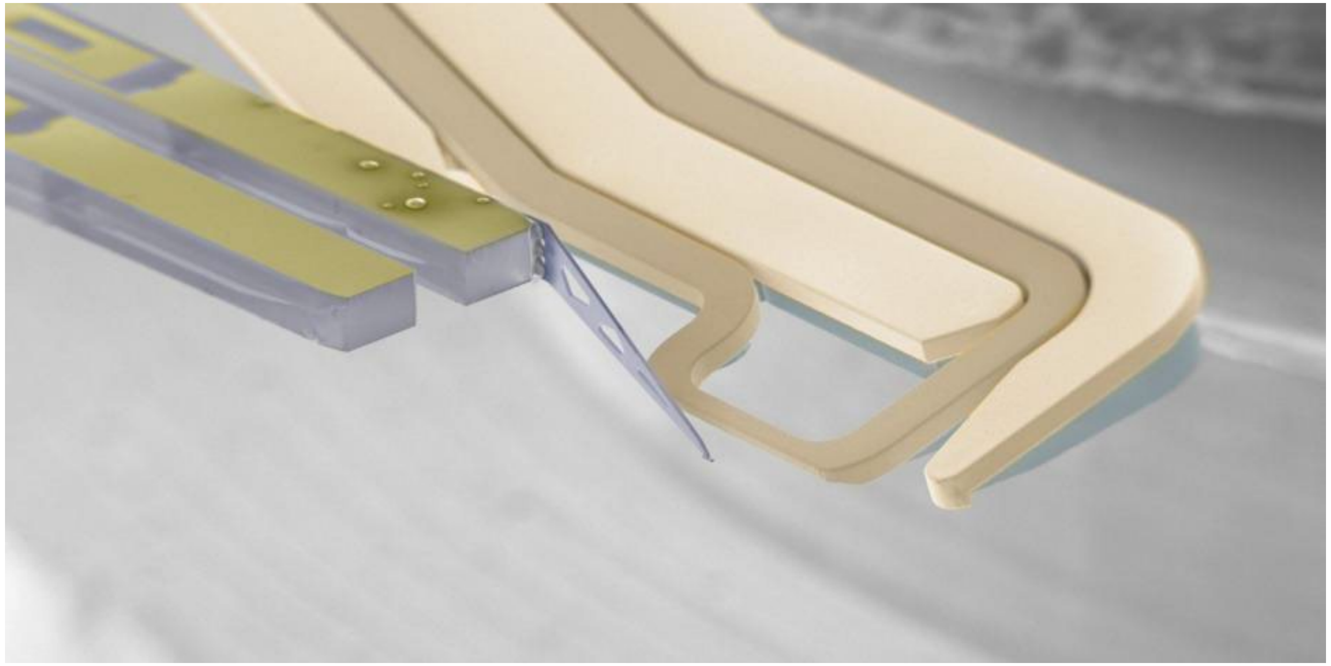
Triplet
singlet
Non-radiative decay
Triplet
Radiative decay
638-800 nm
+1
-1
0
2.87 GHz
RF frequency

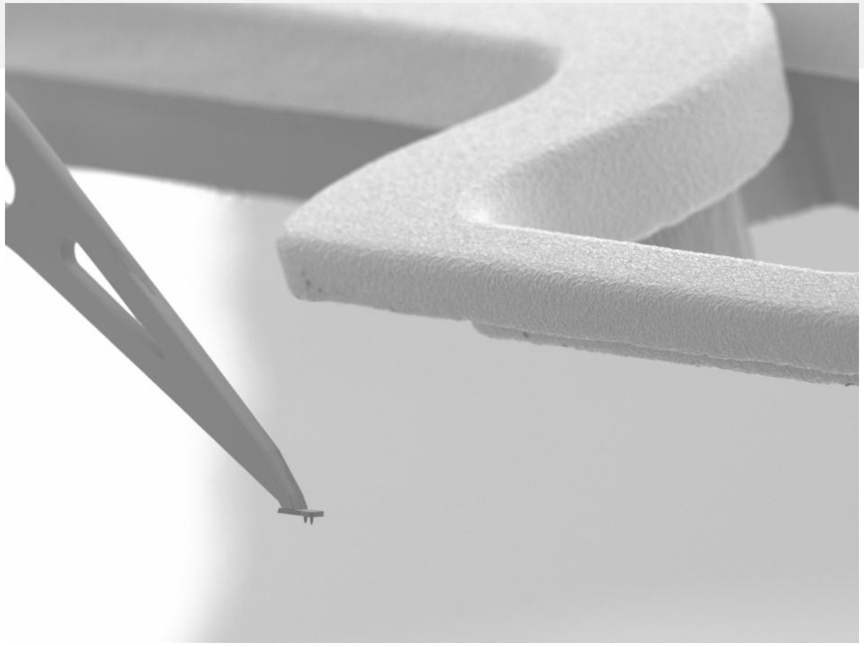


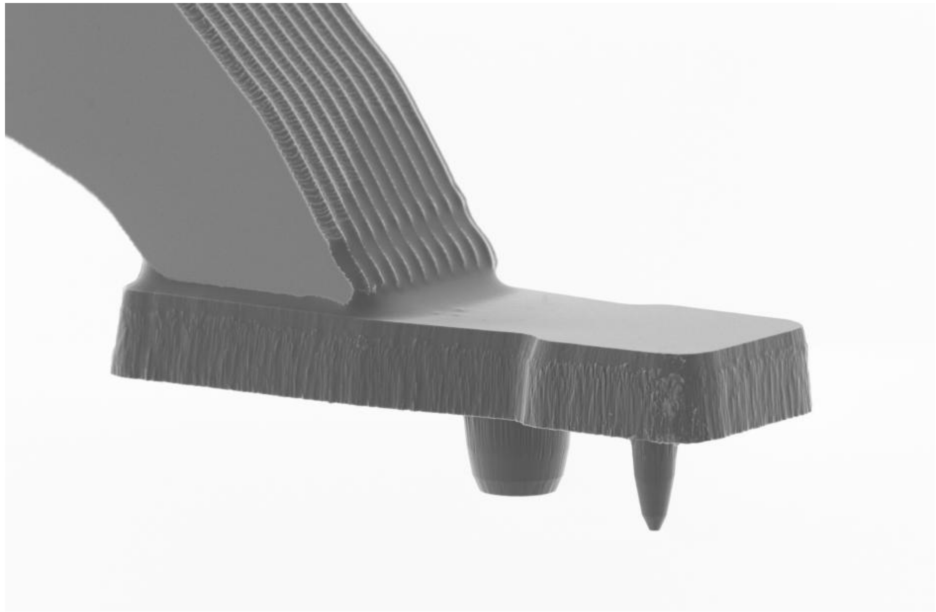
Fluorescence [au]
Frequency [GHz]
2.82
2.87
2.92
28 MHz/mT
2.0 mT
1.5 mT
1.0 mT
0.5 mT
0 mT


To optics
Scanning probe
Diamond tip
V
C
N
NV Center
Magnetic sample surface

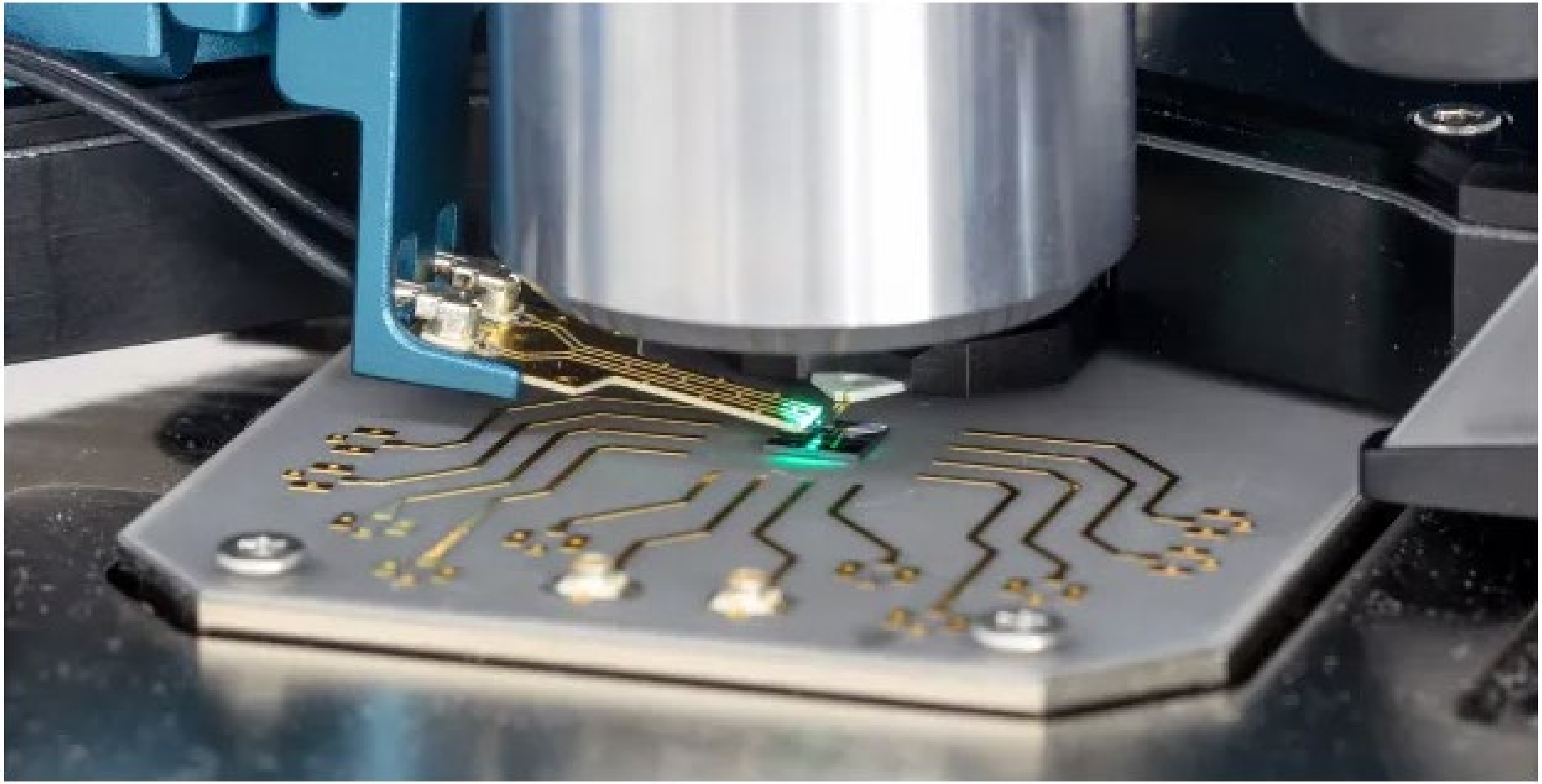

Illumination
Fluorescence
RF signal
N
V
Diamond
NV
Magnetic Sample Surface



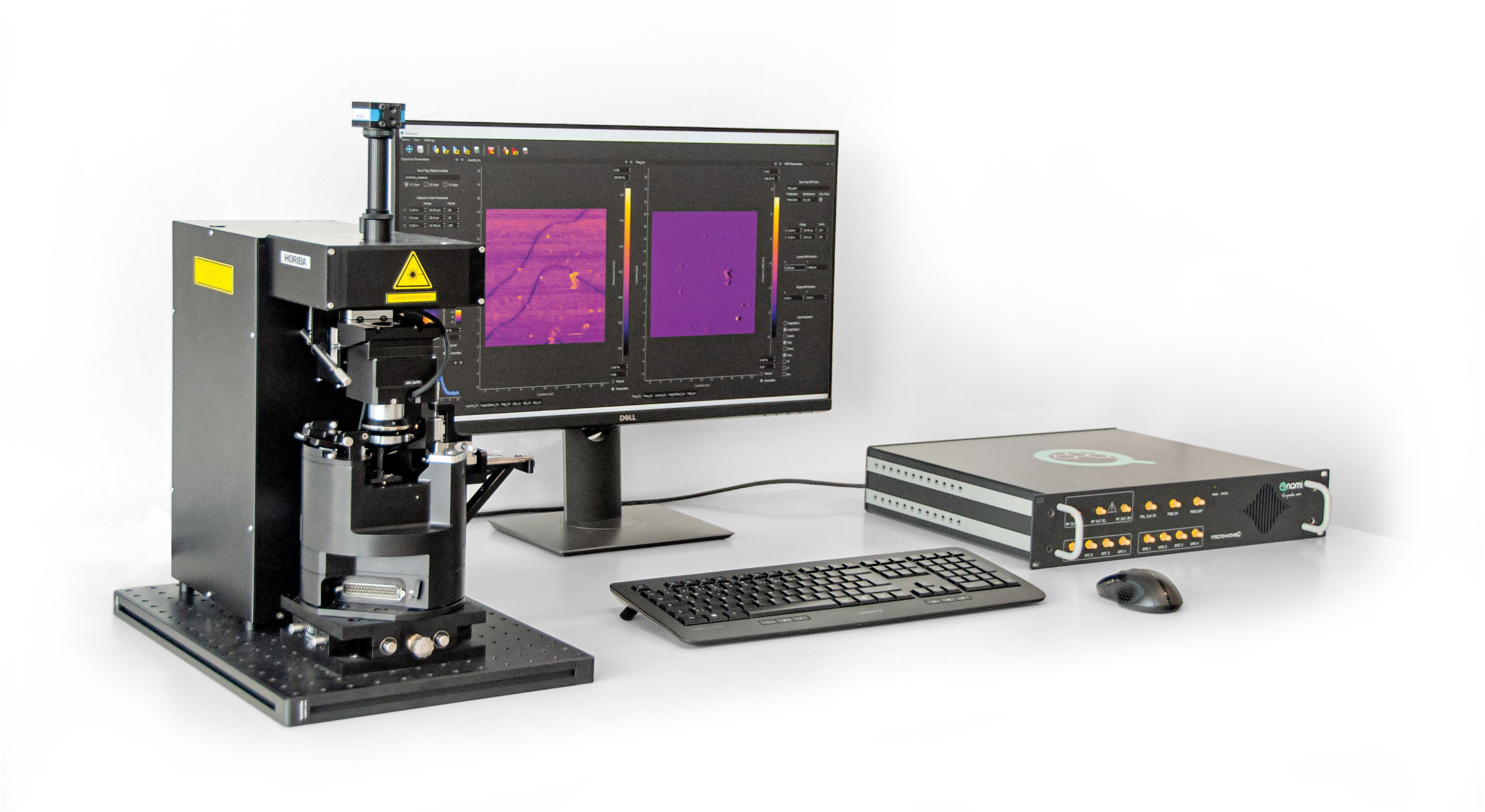




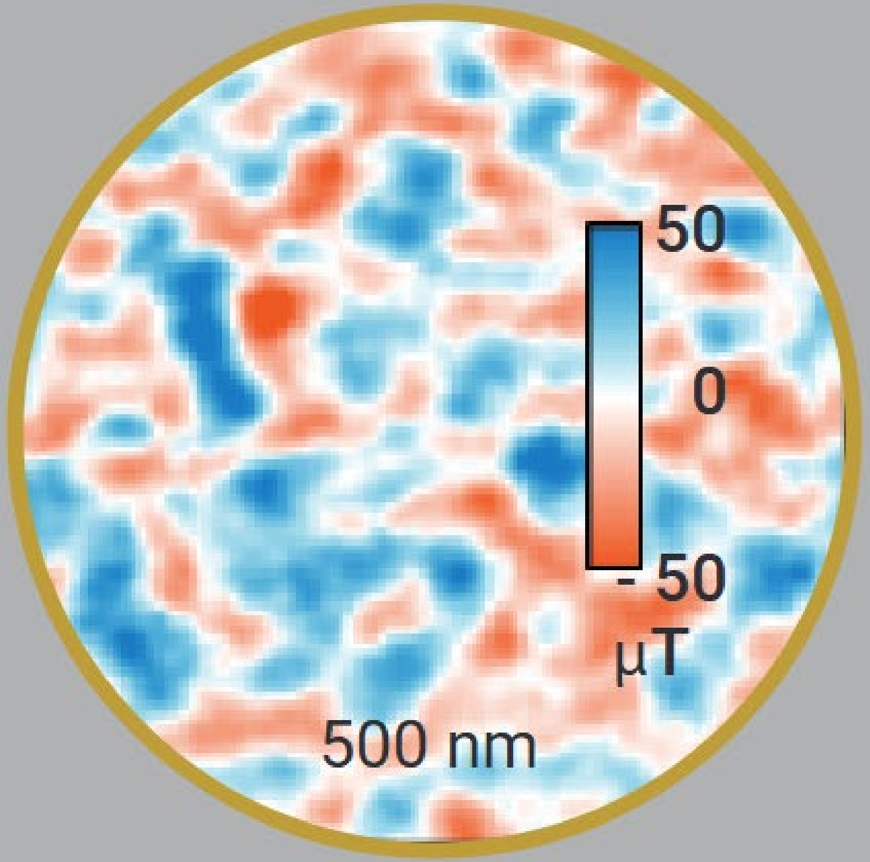
https://qzabre.com



Close-up view of a scanning NV magnetometer with the QZabre sensor in the center, illuminated by green laser light. Microscope objective, microwave antenna and sample holder are also visible.



qnami.ch



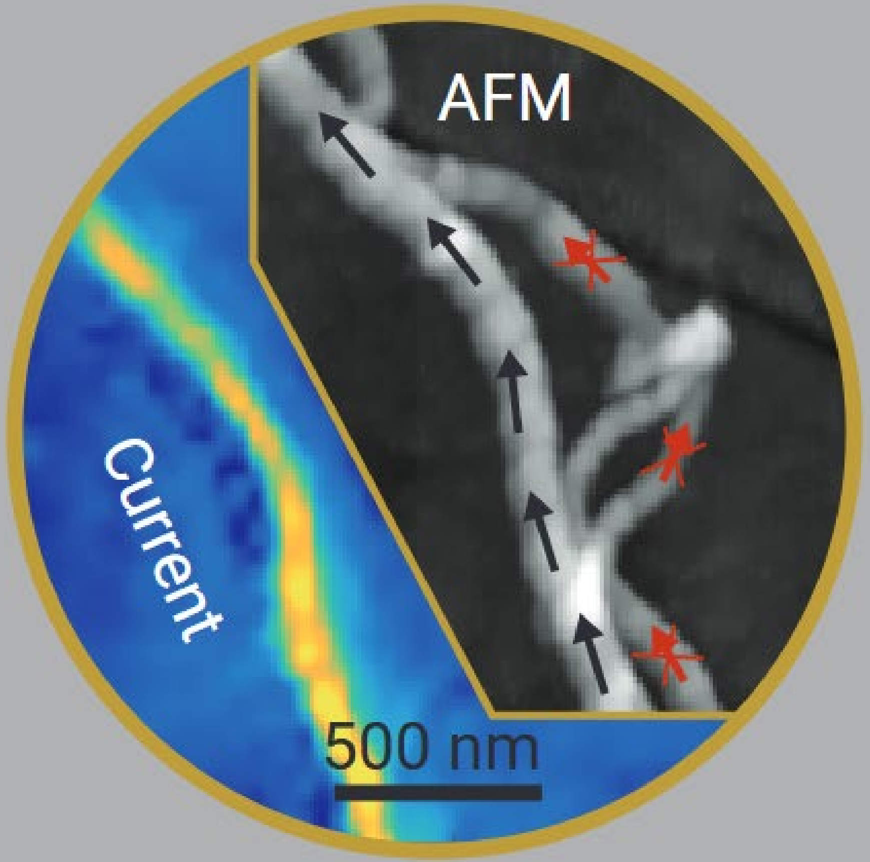


Image Weak Fields

30 nm lateral resolution

1 $\mu T/Hz^{1/2}$ sensitivity

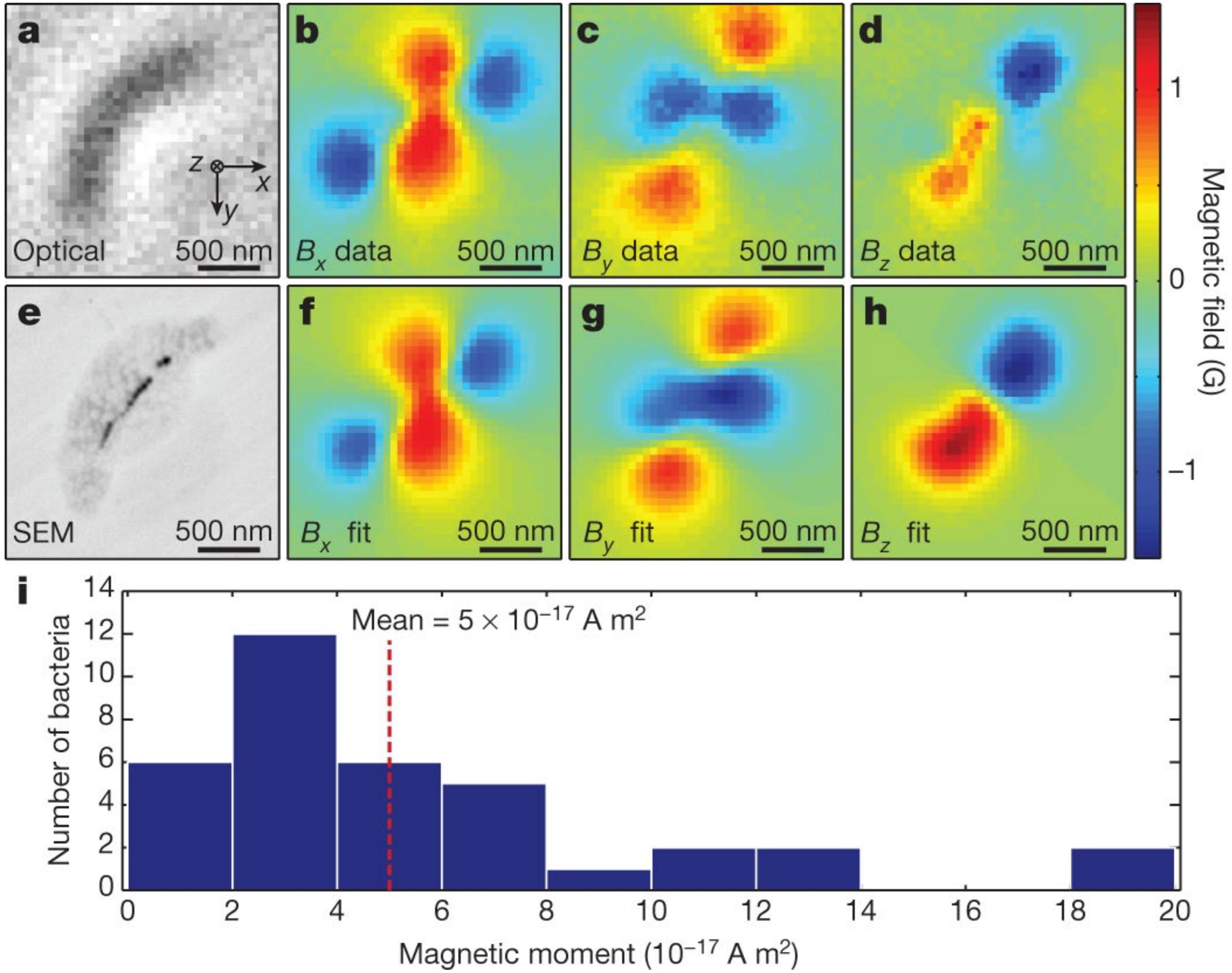
Trace Current Paths

Carbon Nanotube example:

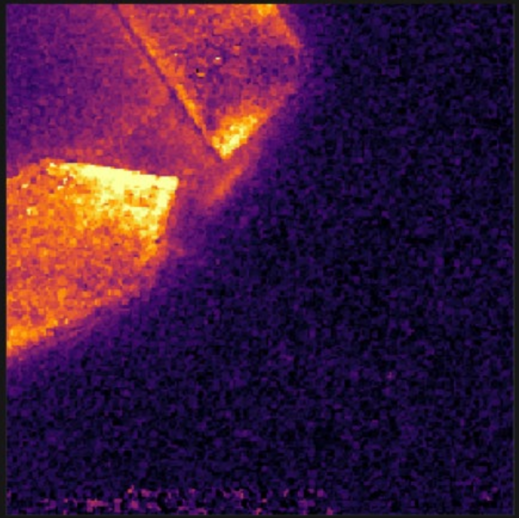
Only one tube is conducting

https://qzabre.com

Determining magnetic moments of individual bacteria from measured magnetic field distributions.

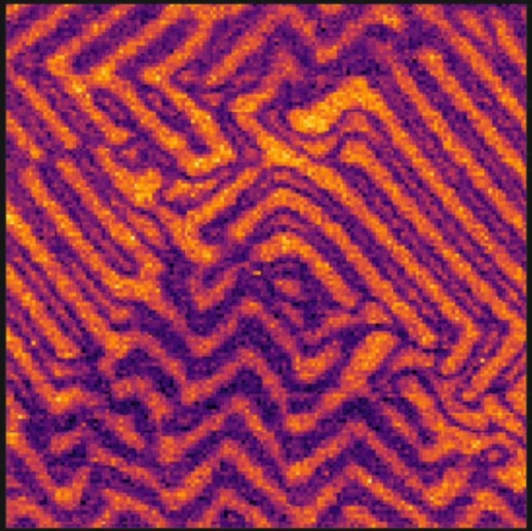


D Le Sage *et al.* *Nature* **496**, 486-489 (2013) doi:10.1038/nature12072



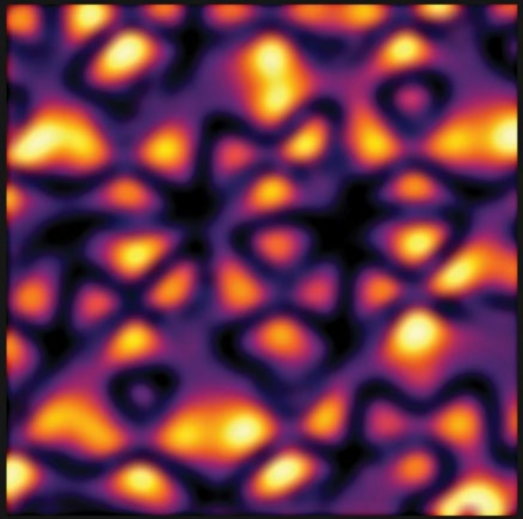
Current Flow in Nanostructures

ODMR contrast given by the local AC stray field
Sample: Graphene, contacted with Ti/Au electrodes



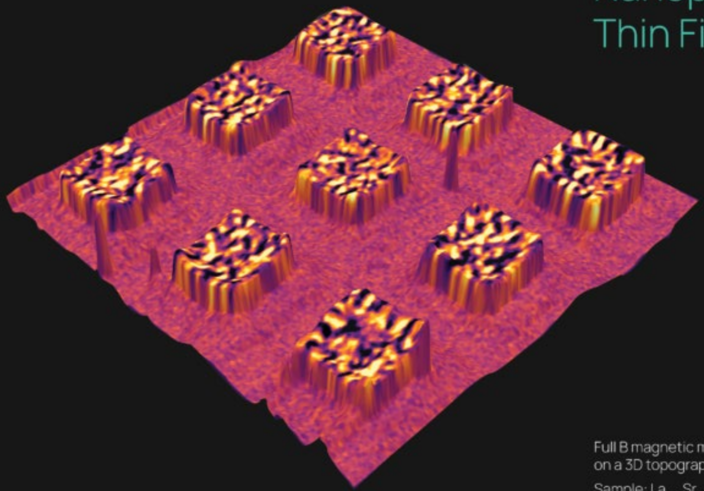
Multiferroics for Spintronics

Single Iso-B map
Sample: $BiFeO_3/DyScO_3$



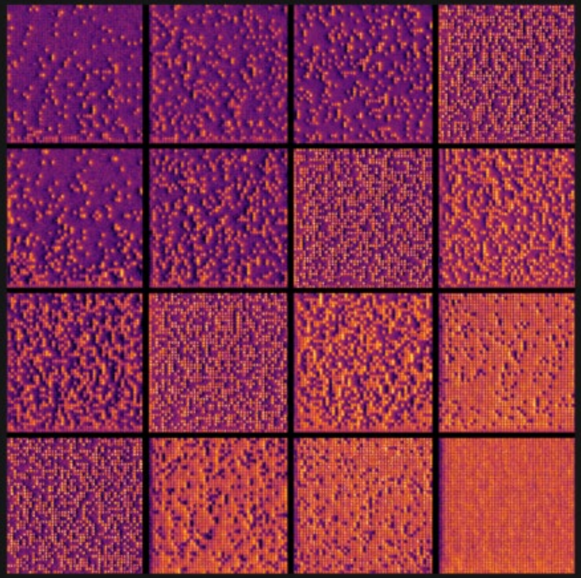
Artificial Spin Ice

Full-B magnetic map
Sample: nanomagnets



Nanopatterned Thin Films

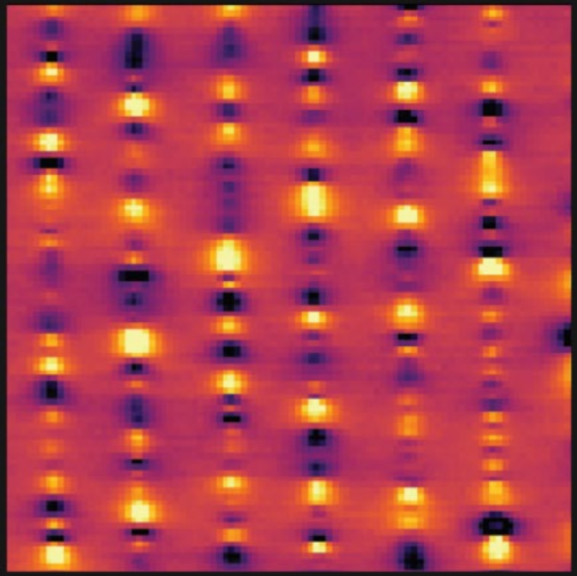
Full B magnetic maps of antiferromagnetic domains projected on a 3D topographic image of differently shaped nanostructures
Sample: $La_{0.67}Sr_{0.33}FeO_3$



MRAM

Grid of magnetic maps taken after applying a magnetic initialization and switching protocol

Sample: STT-MRAM chips of 50 nm diameter, with 200 nm pitch, measured on top of a 200 nm thick encapsulation layer



Magnetic Nanowires

Sample: CoFeB ultrascaled nanowires (6 nm wide)

Full B magnetic map

qnami.ch

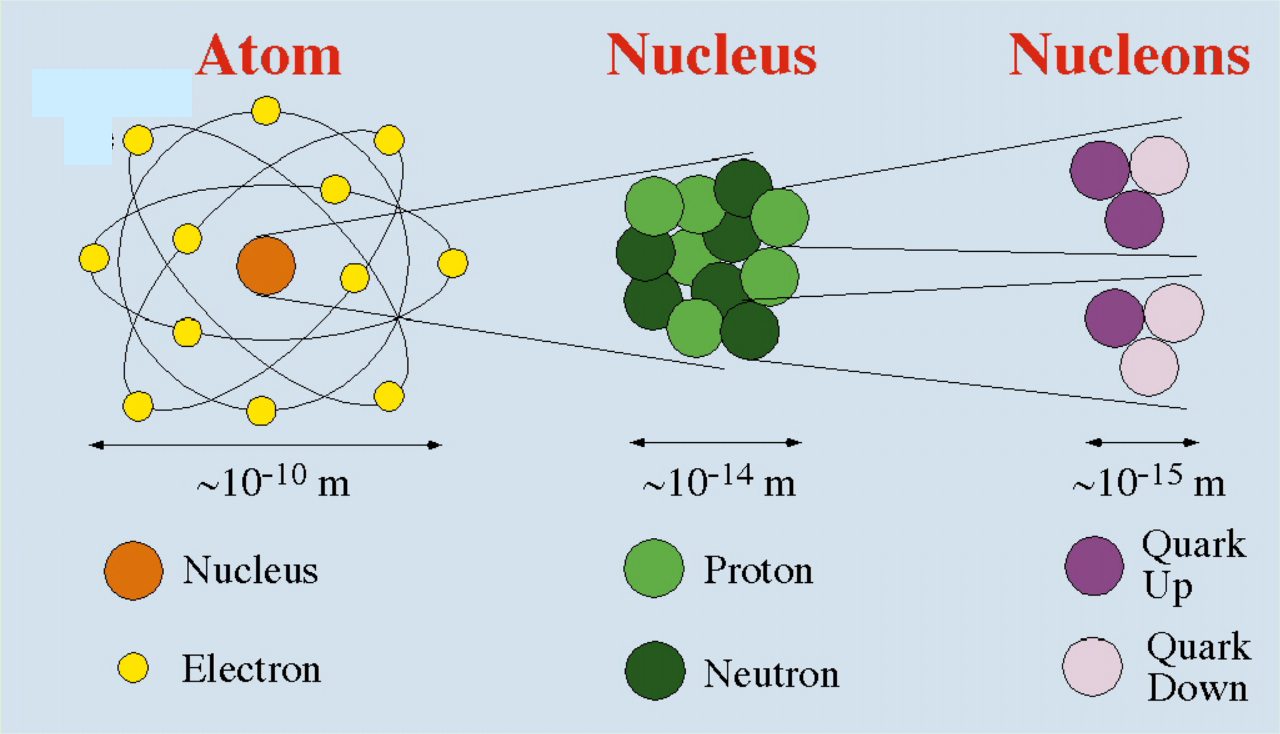
Magnetic Resonance Imaging (MRI)

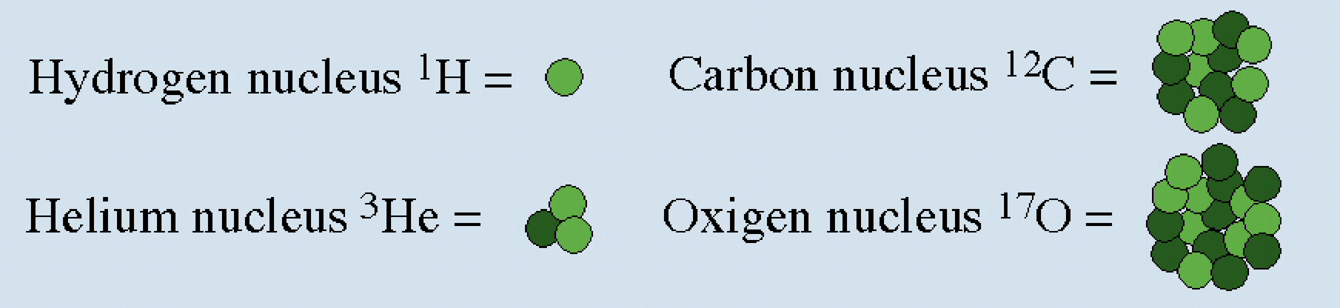
Magnetic Resonance Imaging (MRI)

=

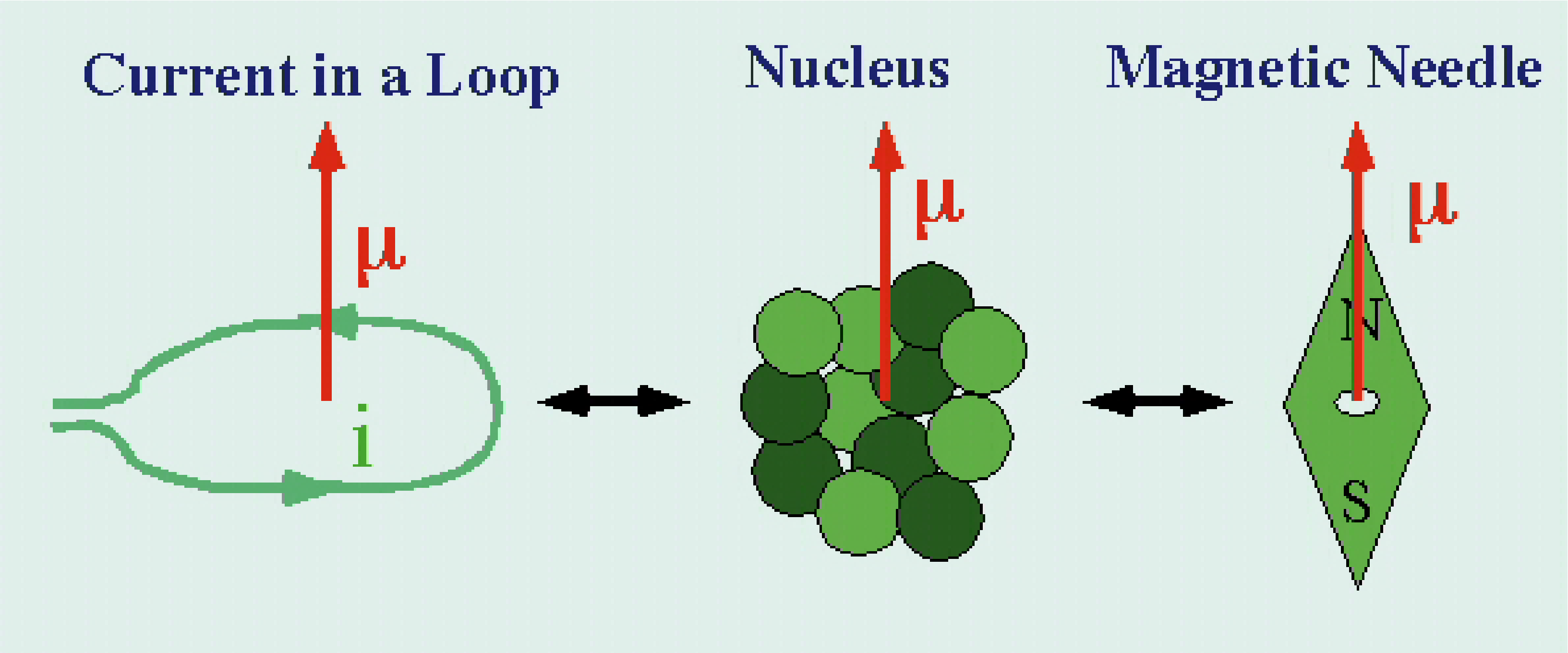
Nuclear Magnetic Resonance Imaging (NMRI)

MRI: …the nucleus…

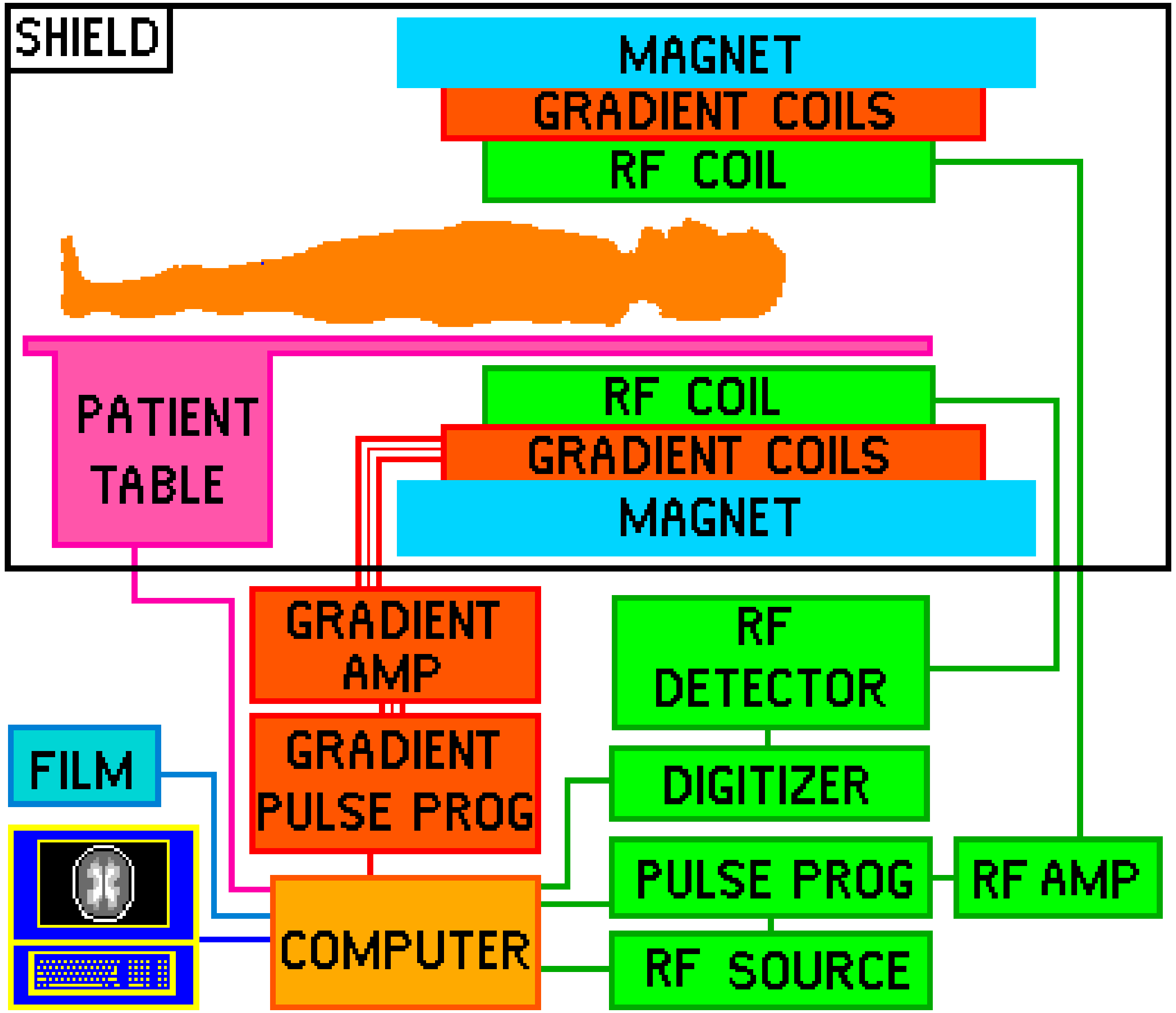




MRI: …the magnetic moment of a nucleus…

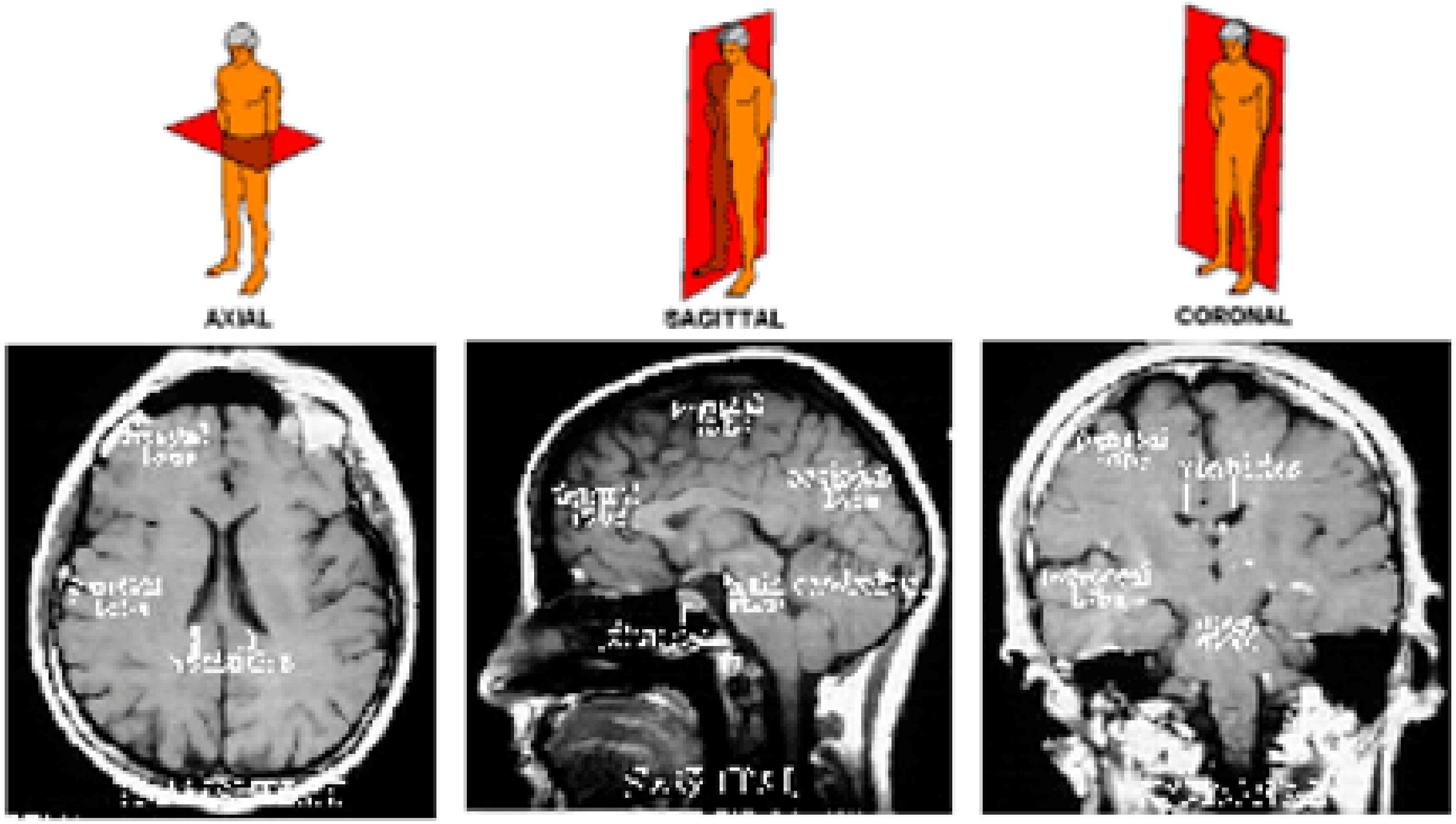


MRI: set-up



Main hardware:
- Magnet
- Gradients coils
- RF detection coil
- Electronics
- Computer

MRI: 3D imaging



MRI: key features

Spatial resolution

- Typical: (1 mm)^2 to (100 μm)^3, best: (3 μm)^3

Strong points

- Non-invasive
- 3D imaging
- Applications in: medicine, biology, physics, chemistry,, …

Weak Points

- Limited spatial resolution
- Long imaging time
- Very expensive set-up (> 500 kCHF)

Applications

- Hospitals:

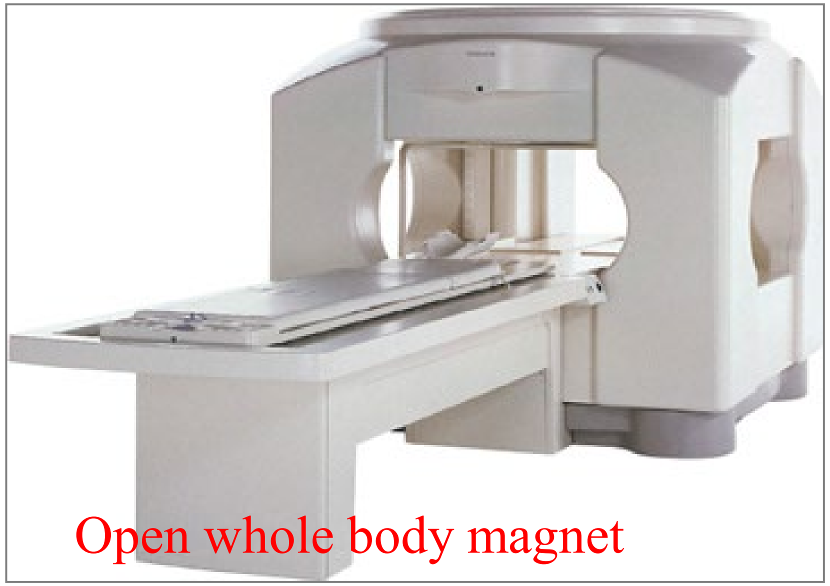
Imaging of whole body (head, brain, arts, lung, heart,…)

- Research:

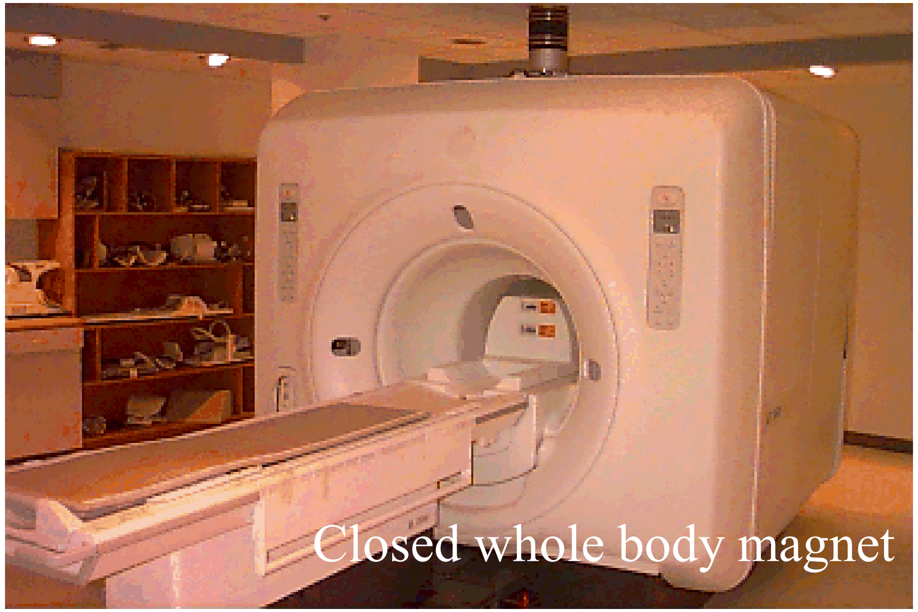
Imaging of plastics, porous rocks, plants, small animals, cells,…

MRI: magnets

Microscopy magnet

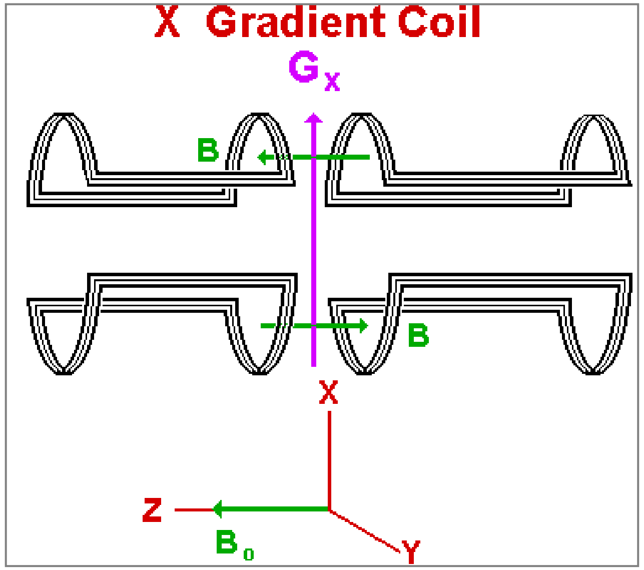


Open whole body magnet

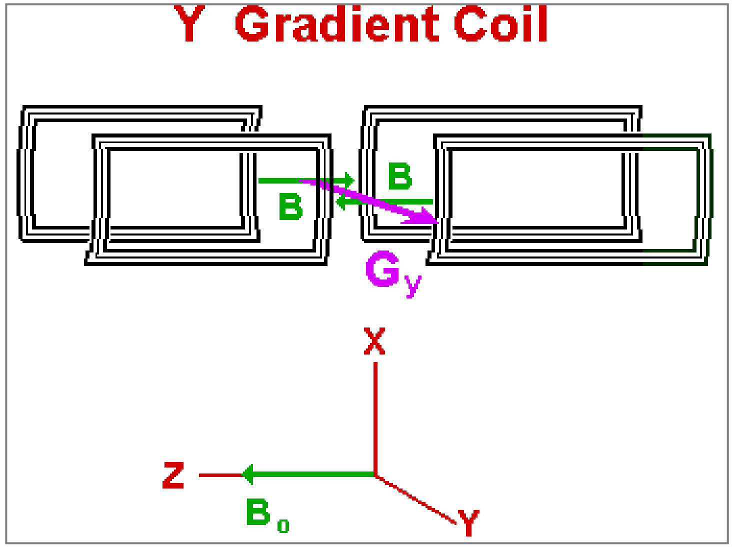


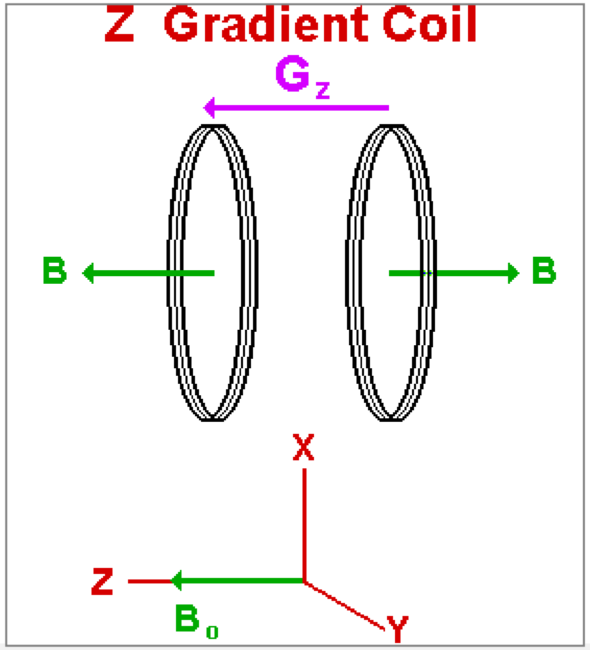
Closed whole body magnet

MRI: gradient coils

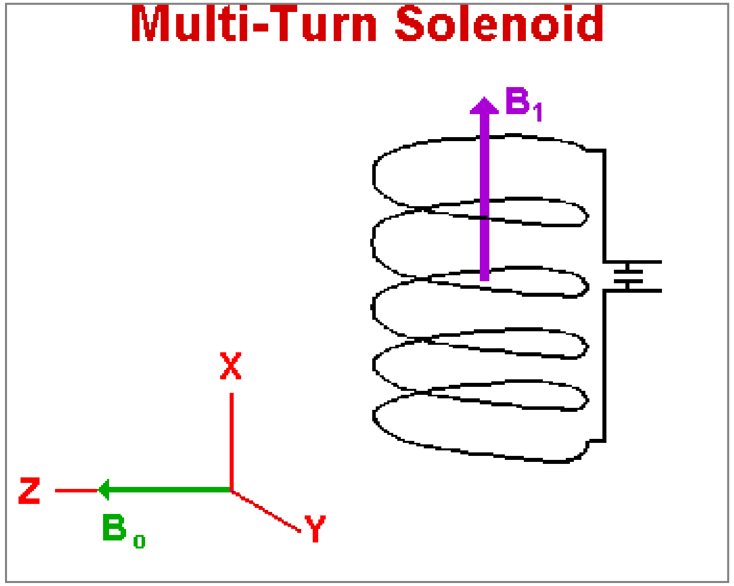


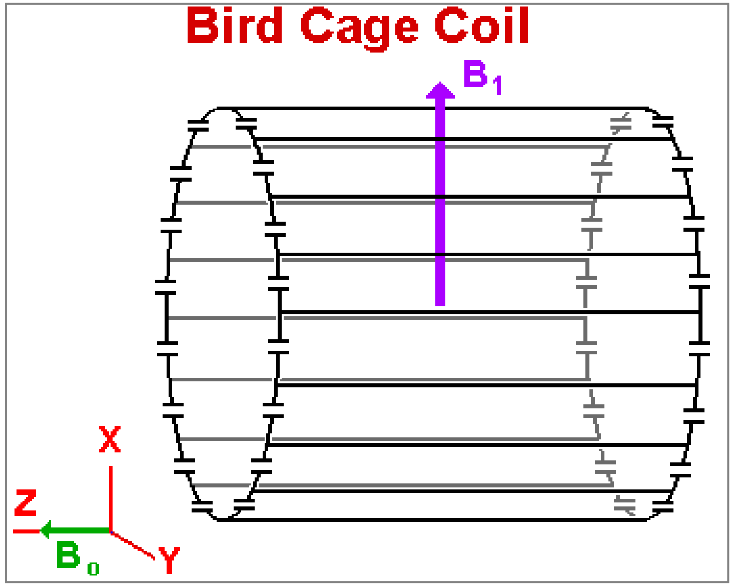
The gradient coils produce linear gradients



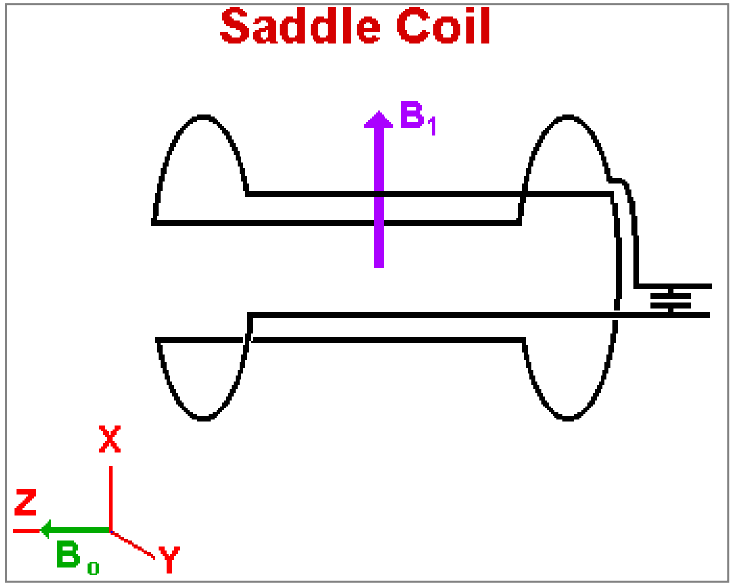


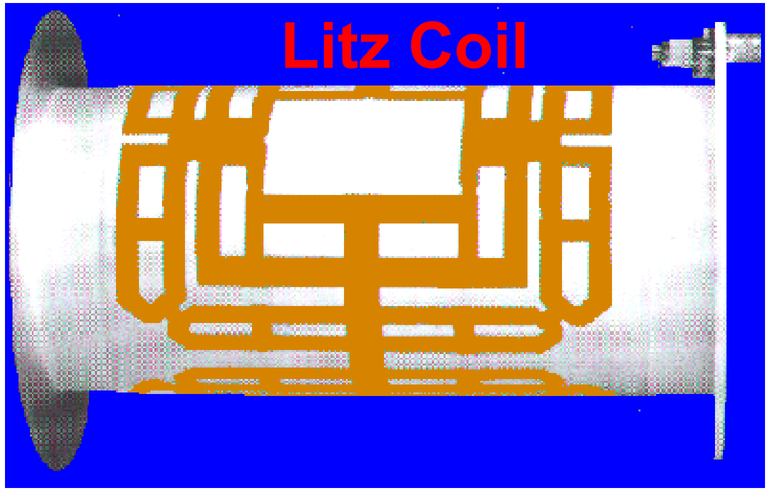
MRI: RF coils



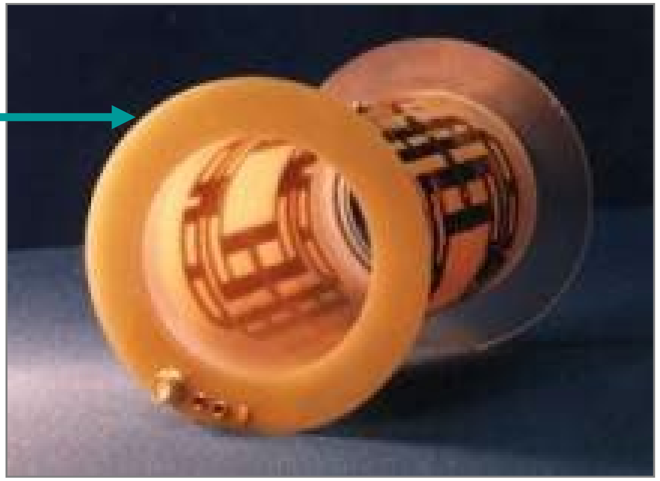


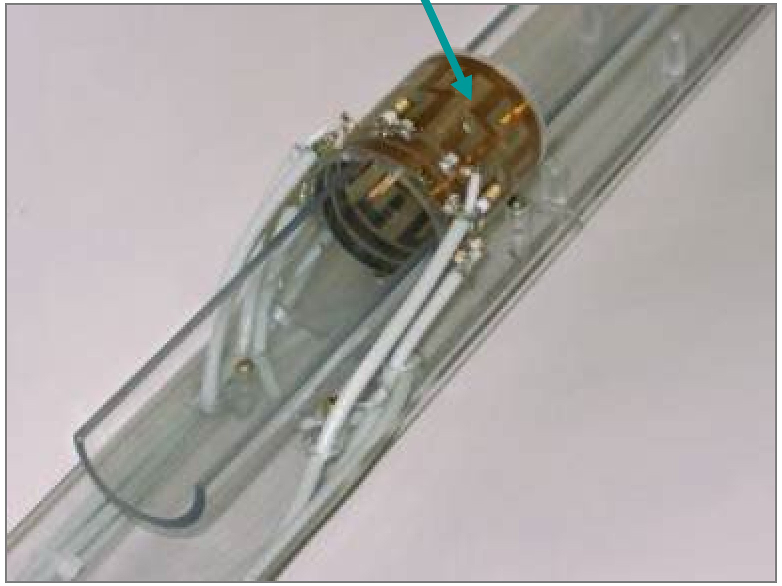
The RF coils produce the RF excitation field and detect the RF NMR signal.





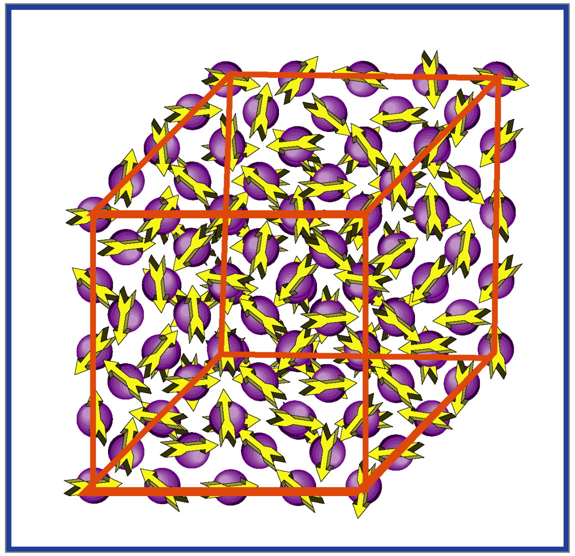
RF coils (Litz coils)

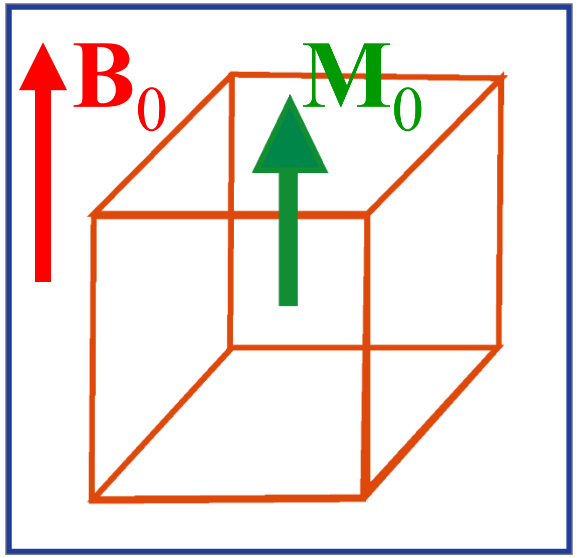




Gradient coils assembly

MRI: basic concepts





$$\mathbf{M}_0 = \chi \mathbf{H} \cong \frac{\chi}{\mu_0} \mathbf{B}_0$$

$$\chi = \mu_0 \frac{N\mu^2}{3kT}$$

Electronic paramagnetism

$$\chi \approx 10^{-3} \div 10^{-6}$$

Origin: magnetic moment of of molecules, atoms, ions.

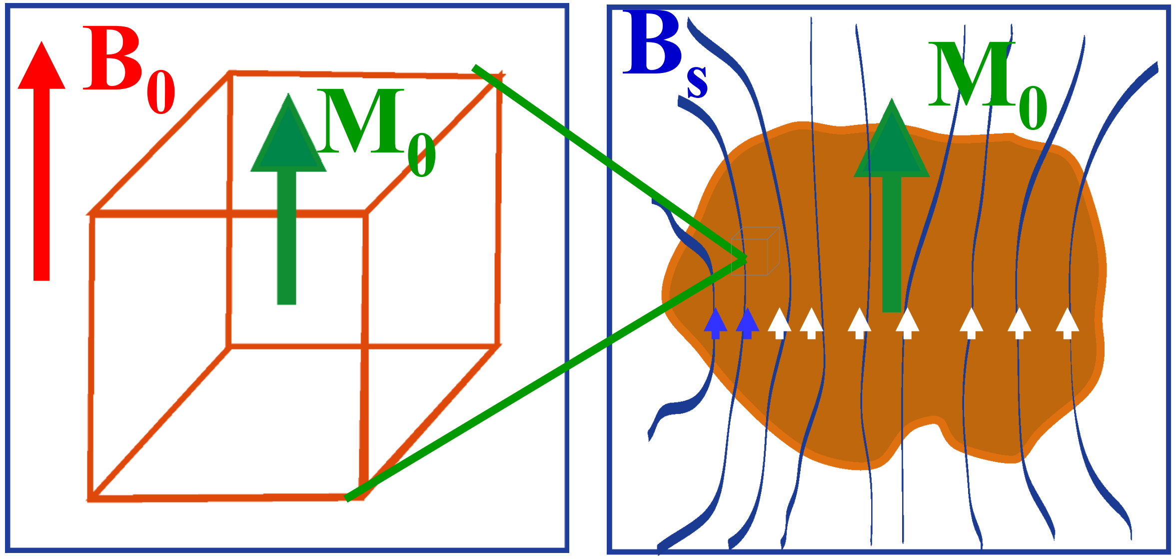
Ex: O_2, Cu^{2+},…

Nuclear paramagnetism

$$\chi \approx 10^{-10} \div 10^{-5}$$

Origin: magnetic moment of the nuclei

Ex.: ^{1}H, ^{3}He, ^{13}C, ^{19}F,…



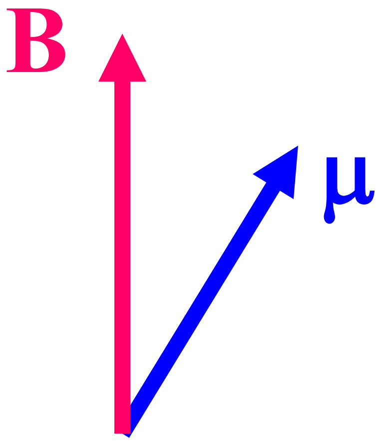
$$\chi << 1$$
$$\Rightarrow$$
$$\mathbf{B}_s \cong \chi \mathbf{B}_0$$

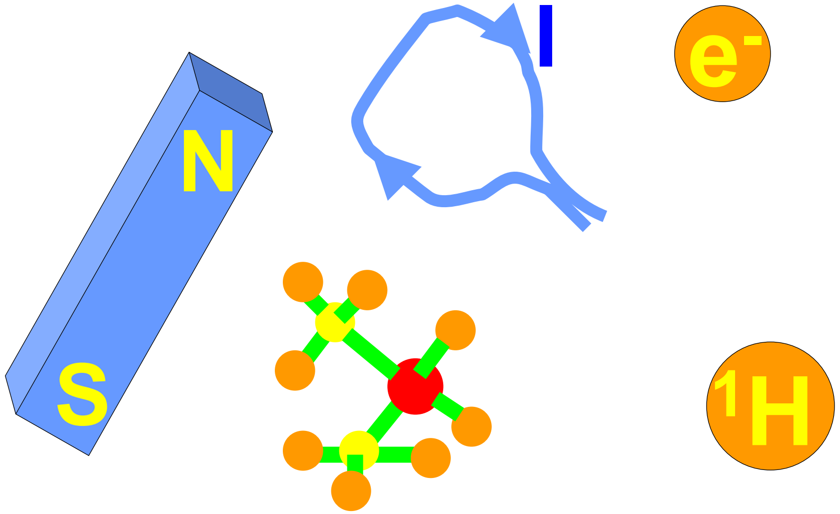
Example:

^1H in liquid H_20
(B_0=1 T, T=300 K)

$$\chi \cong 10^{-9} \Rightarrow B_s \cong 1\,\mathrm{nT}$$

Magnetic moment in a magnetic field

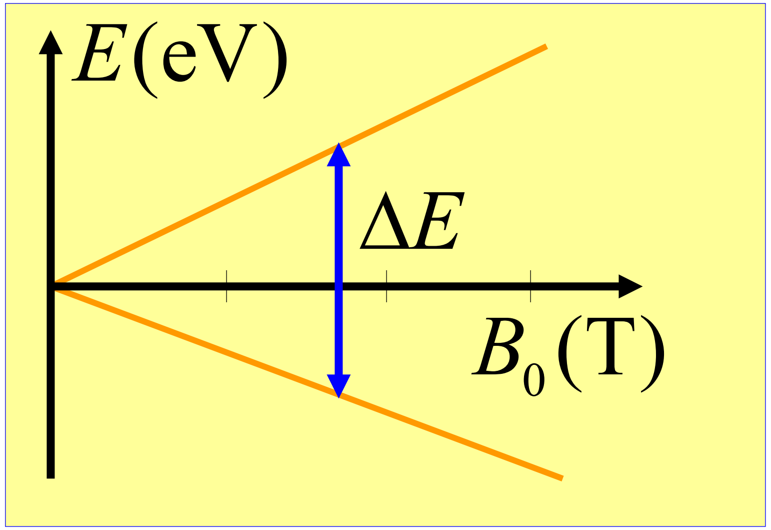




$$\begin{cases} \boldsymbol{\tau} = \boldsymbol{\mu} \wedge \mathbf{B} \\ \boldsymbol{\tau} = \dfrac{d\mathbf{L}}{dt} \\ \boldsymbol{\mu} = \gamma\, \mathbf{L} \end{cases} \quad \Longrightarrow \quad \frac{d\boldsymbol{\mu}}{dt} = \gamma\, \boldsymbol{\mu} \wedge \mathbf{B}$$

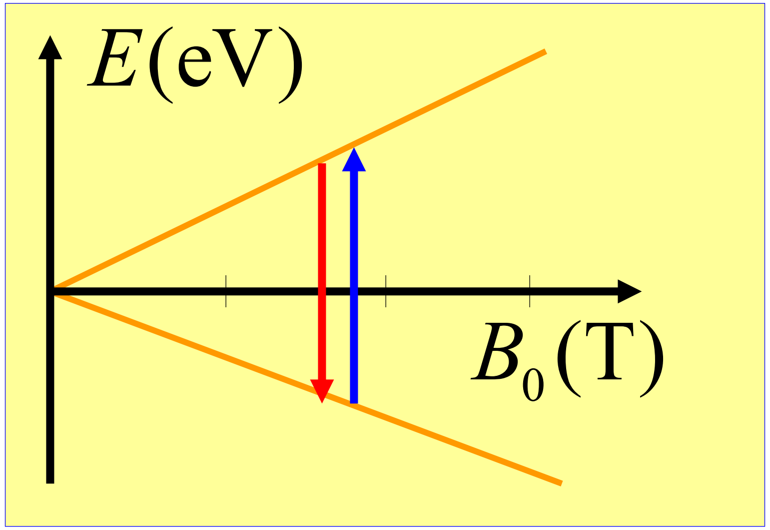
Static magnetic field

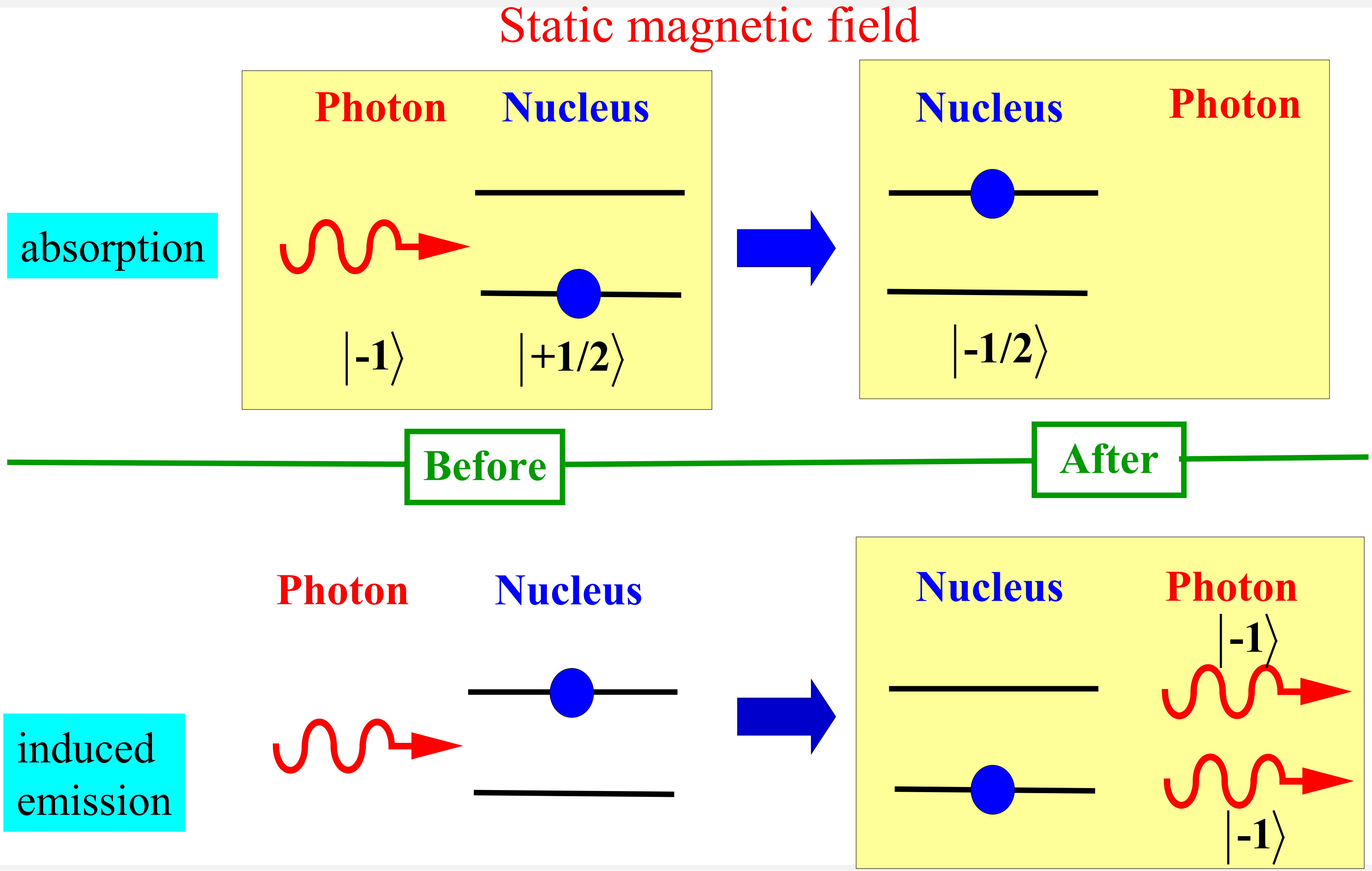
$$\begin{cases} H = -\boldsymbol{\mu} \cdot \mathbf{B}_0 \\ \boldsymbol{\mu} = \gamma\, \mathbf{L} = \gamma\, \hbar\, \mathbf{I} \end{cases} \Rightarrow \Delta E = \gamma\, \hbar\, B_0$$



Static + RF magnetic field

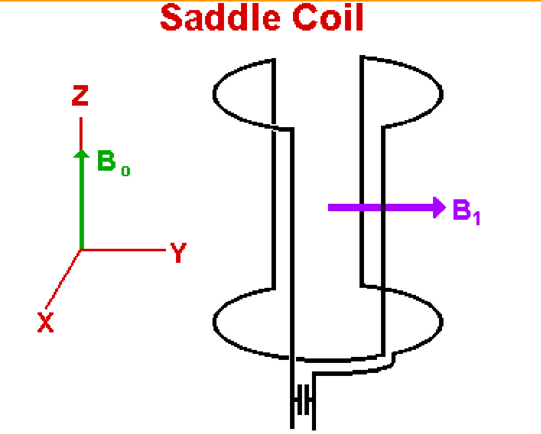
$$\begin{cases} \mathbf{B} = \mathbf{B}_0 + \mathbf{B}_1 \cos \omega t \\ \hbar\omega \cong \Delta E \end{cases} \Rightarrow \begin{cases} \text{induced emission} \downarrow \\ \text{absorption} \uparrow \end{cases}$$

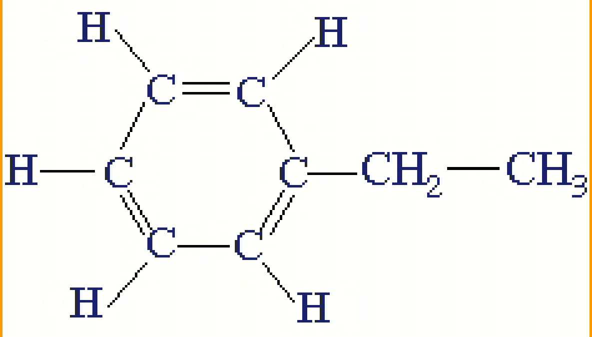


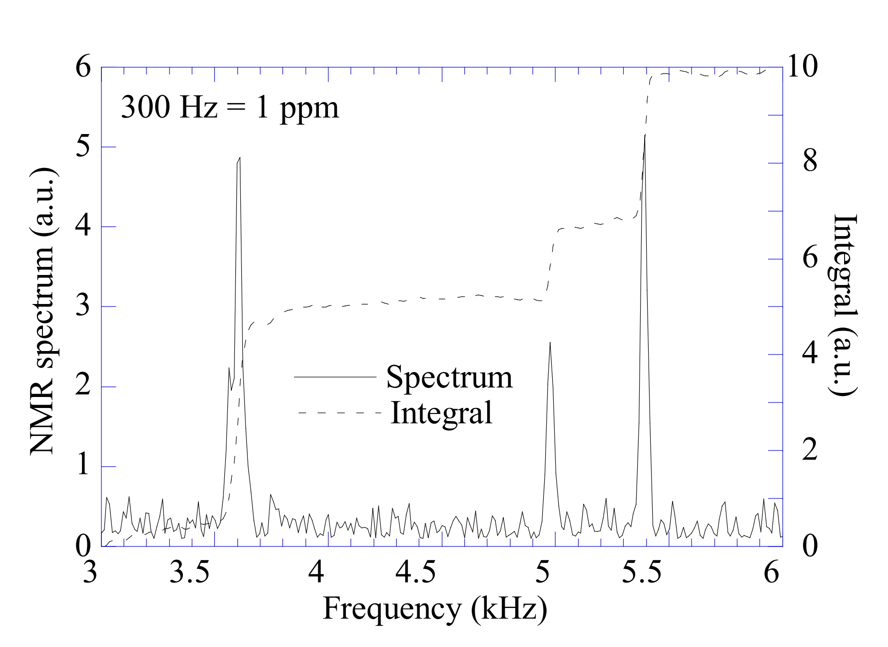

Static magnetic field
absorption
Photon
Nucleus
|-1⟩
|+1/2⟩
Nucleus
Photon
|-1/2⟩
Before
After
induced
emission
Photon
Nucleus
Nucleus
Photon
|-1⟩
|-1⟩

MRI: NMR spectroscopy

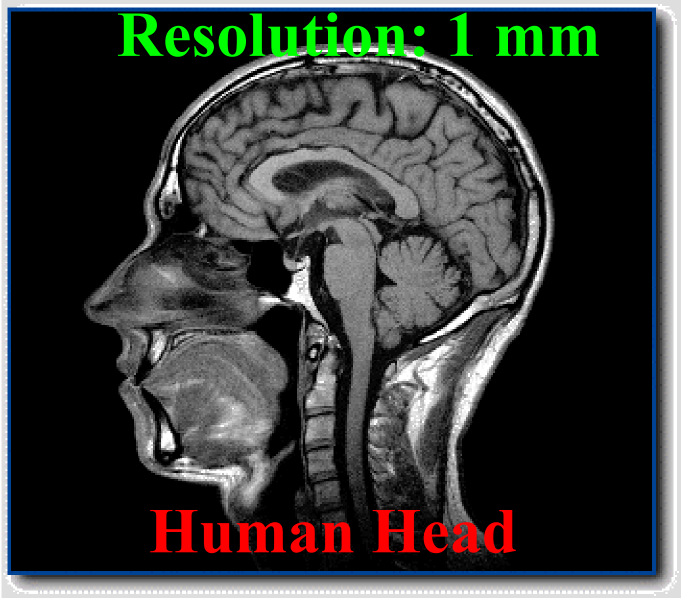
Spectroscopy magnet

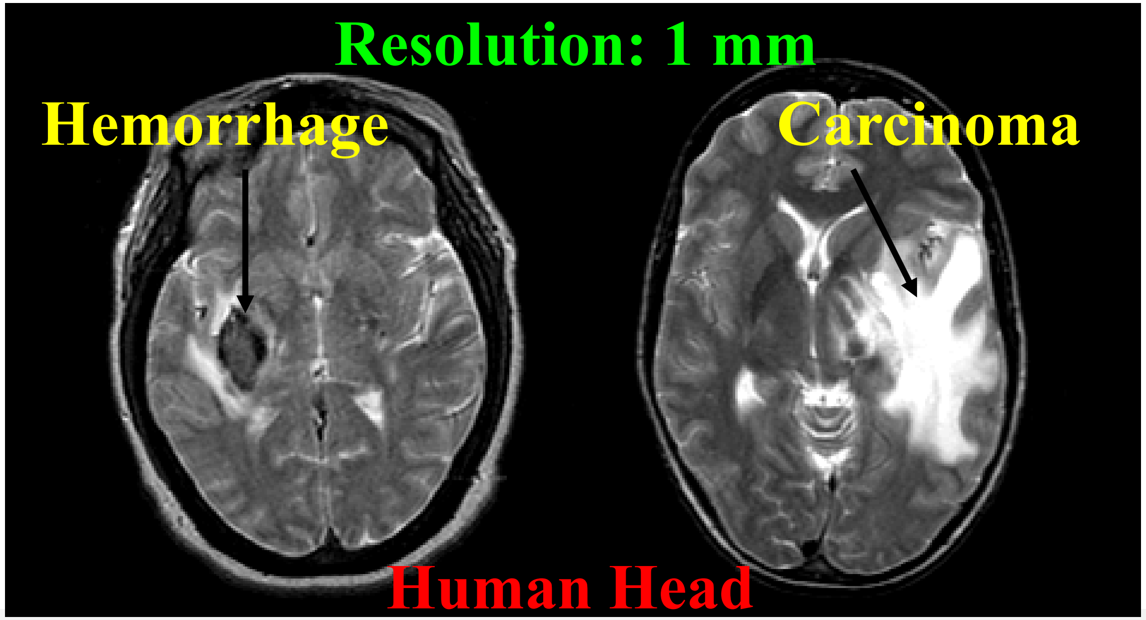


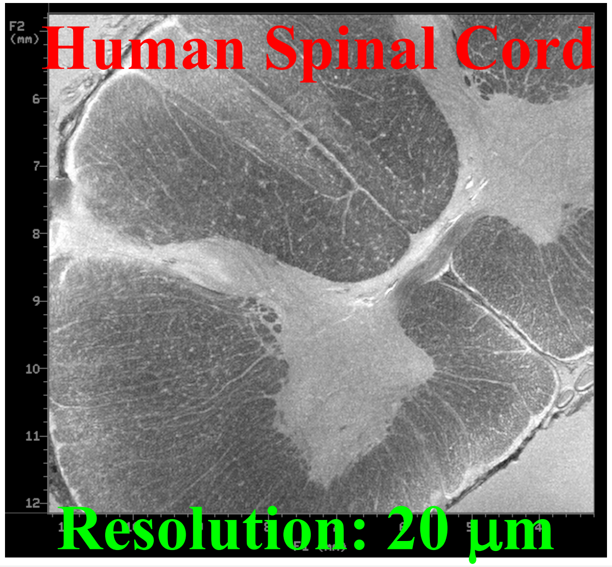




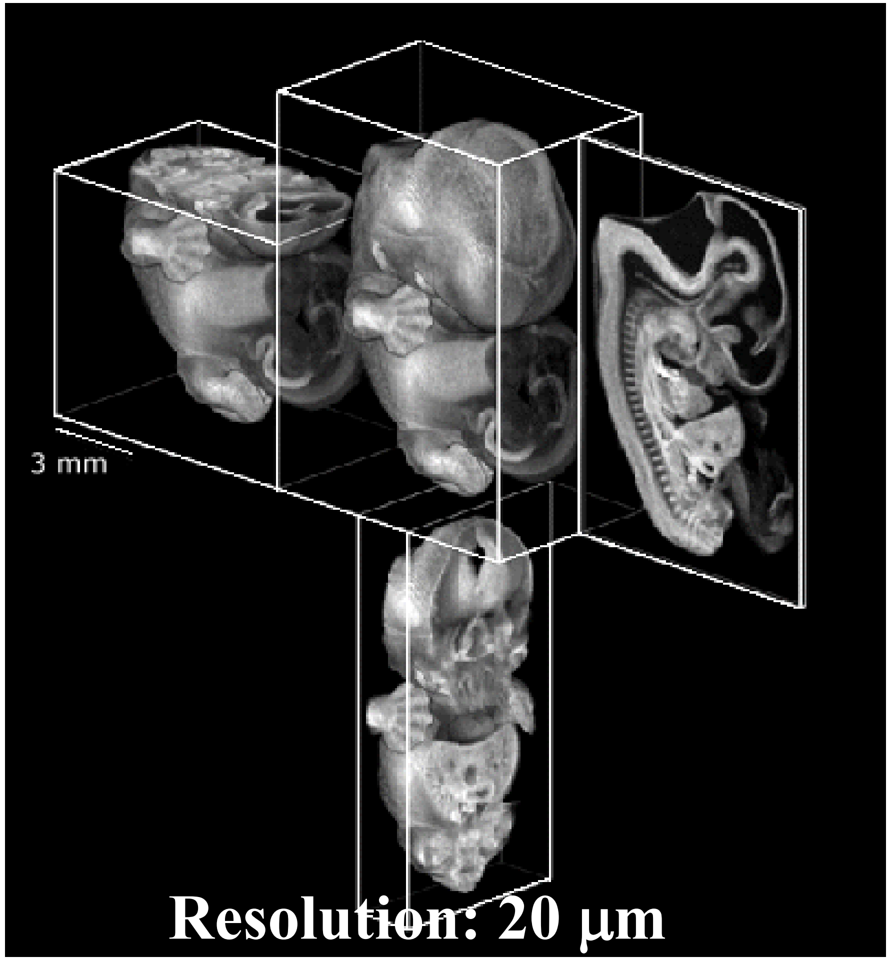
MRI: NMR imaging (MRI)



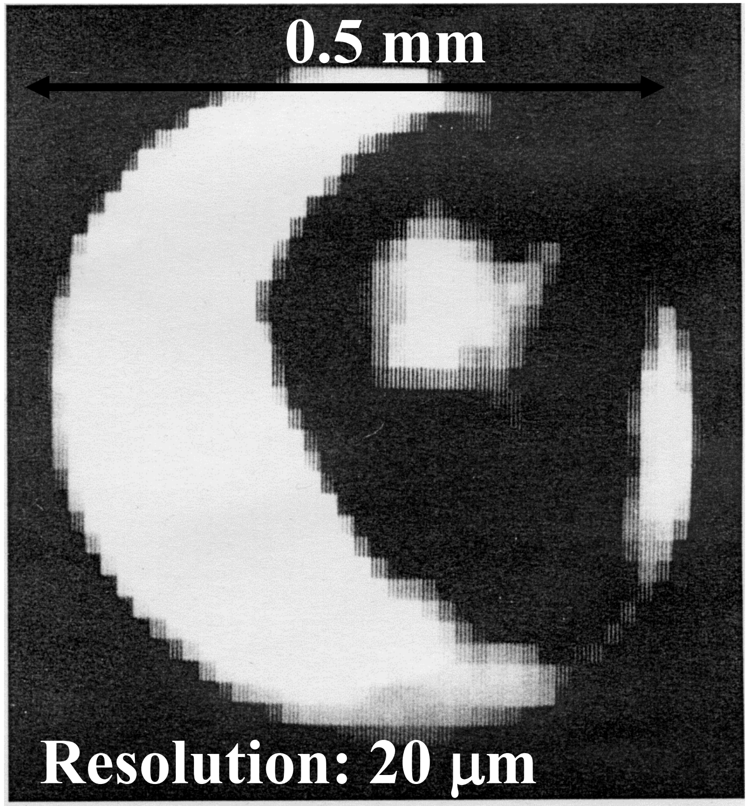




MRI: NMR microscopy (MRI microscopy)

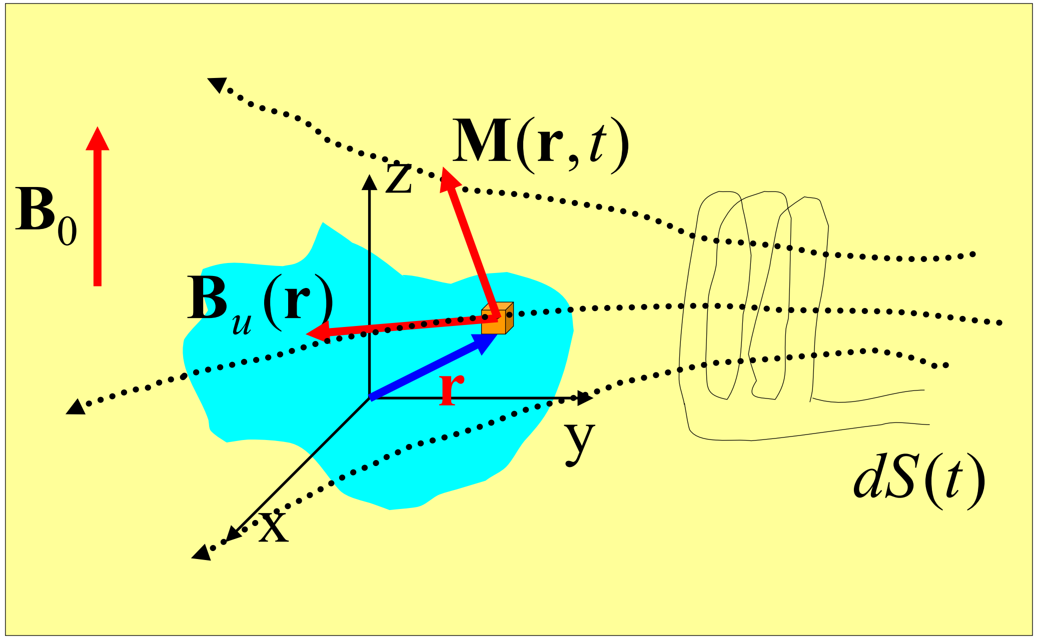


47-days Human Embryo



Single neuron

MRI: basic concepts



$\mathbf{M}(\mathbf{r},t)$: nuclear magnetization (A/m)

$\mathbf{B}_u(\mathbf{r})$: coil unitary field (A/T)

$dS(t)$: induced voltage (V)

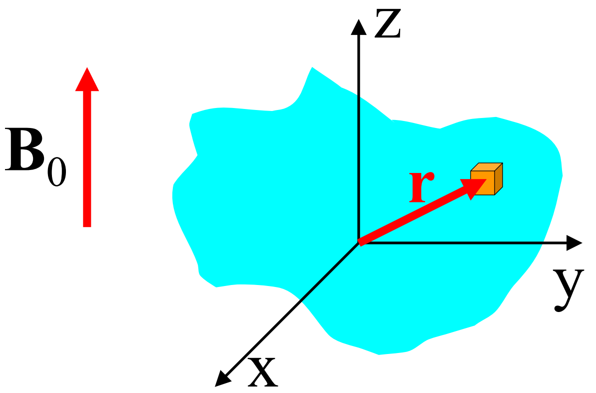
$\rho(\mathbf{r})$: spin density (m^{-3})

Reciprocity principle: $dS(t,\mathbf{r}) = -\frac{d}{dt}(\mathbf{B}_u(\mathbf{r})\cdot\mathbf{M}(\mathbf{r},t))d\mathbf{r}$

For $B_u(\mathbf{r}) // \mathbf{M}(\mathbf{r},t)$

Since $M(\mathbf{r}) \propto \rho(\mathbf{r})$:

$$dS(t,\mathbf{r}) \propto \omega(\mathbf{r})\rho(\mathbf{r})B_u(\mathbf{r})\exp\left(i\omega(\mathbf{r})t\right)d\mathbf{r}$$



$$\omega(\mathbf{r}) = \gamma\left|\mathbf{B}(\mathbf{r})\right|$$

$$\mathbf{B}(\mathbf{r}) = \mathbf{B}_0 + [\mathbf{G}]\cdot\mathbf{r}$$

$[\mathbf{G}]$: linear gradient tensor

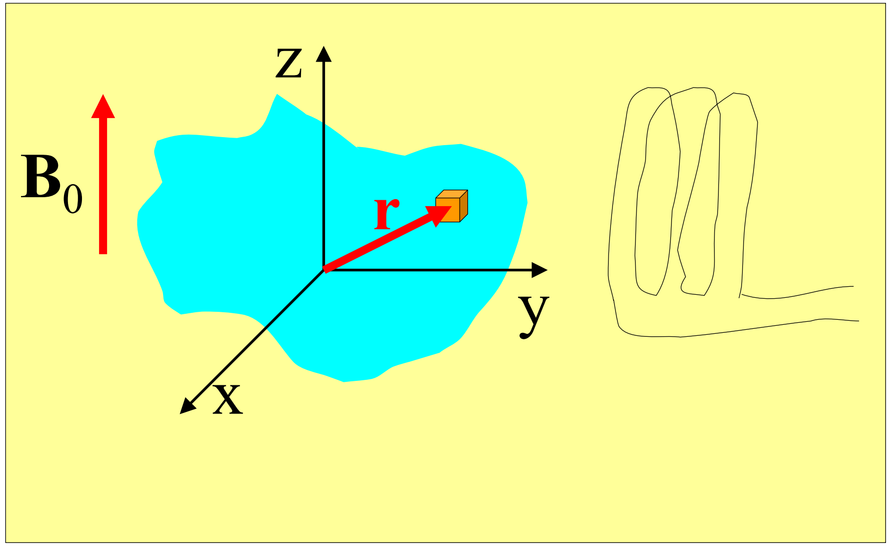
$$[\mathbf{G}] \equiv \begin{bmatrix} G_{xx} & G_{xy} & G_{xz} \\ G_{yx} & G_{yy} & G_{yz} \\ G_{zx} & G_{zy} & G_{zz} \end{bmatrix}$$

For $\mathbf{B}_0=(0, 0, B_0)$ and $B_0 >> G_{ij}$

$$\omega(\mathbf{r}) \cong \gamma B_0 + \gamma\left(G_x x + G_y y + G_z z\right)$$

where $\mathbf{G}$:

$$\mathbf{G} \equiv \left(G_{zx}, G_{zy}, G_{zz}\right) \equiv \left(G_x, G_y, G_z\right)$$



$$\omega(\mathbf{r}) \cong \gamma B_0 + \gamma \mathbf{G} \cdot \mathbf{r}$$

$$dS(t) \propto \rho(\mathbf{r}) d\mathbf{r} \exp(i\omega(\mathbf{r})t)$$

Consequently: $$dS(t) \propto \rho(\mathbf{r}) \exp(i\gamma \mathbf{G} \cdot \mathbf{r})t) d\mathbf{r}$$

The total signal
the signal induced in the coil is:

$$S(t) \propto \iiint \rho(\mathbf{r}) \exp(i\gamma \mathbf{G} \cdot \mathbf{r}t) d\mathbf{r}$$

$$S(t) = \iiint \rho(\mathbf{r}) \exp(i\gamma \mathbf{G} \cdot \mathbf{r}t) d\mathbf{r}$$

Defining $\mathbf{k} \equiv \frac{1}{2\pi} \gamma \mathbf{G} t$

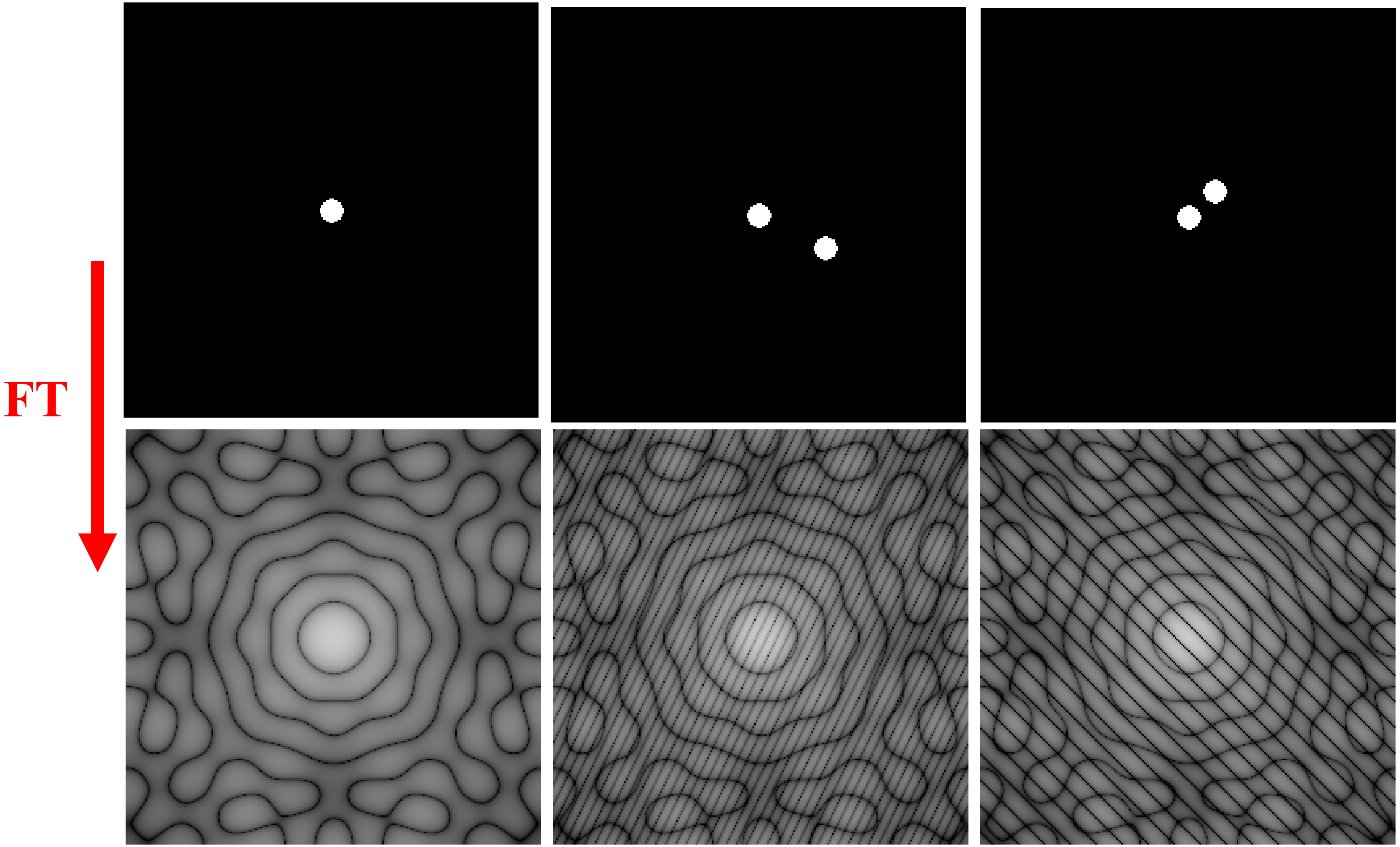
$$S(\mathbf{k}) = \iiint \rho(\mathbf{r}) \exp(i2\pi \mathbf{k} \cdot \mathbf{r}) d\mathbf{r}$$

$$\rho(\mathbf{r}) = \iiint S(\mathbf{k}) \exp(-i2\pi \mathbf{k} \cdot \mathbf{r}) d\mathbf{k}$$

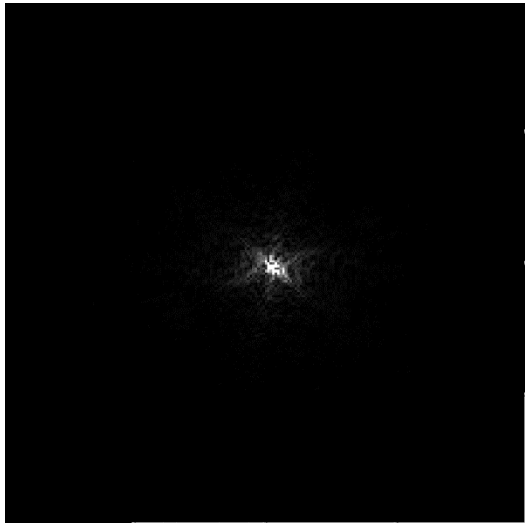
Fundamental relationships in NMR imaging

Measuring $S(\mathbf{k})$ it is possible to compute, by Fourier transform, the spin density $\rho(\mathbf{r})$, i.e. **the NMR image**.

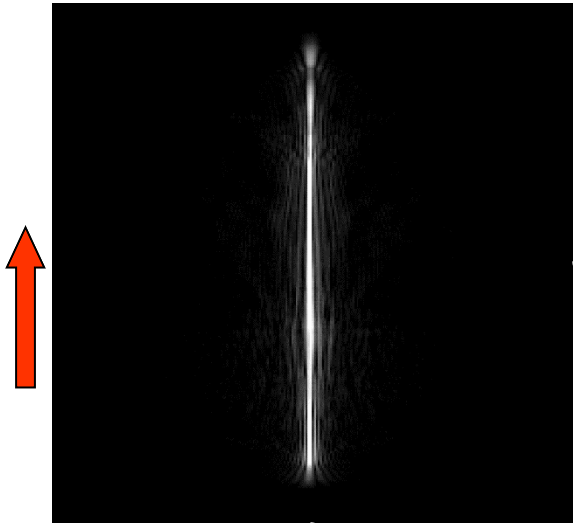
(mathematical example)



Raw data in **k**-space (256x256 points)

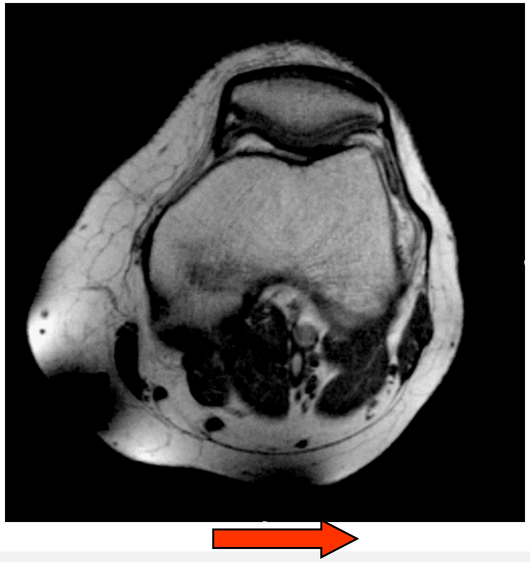


1st Fourier transform



Human Knee

2nd Fourier transform

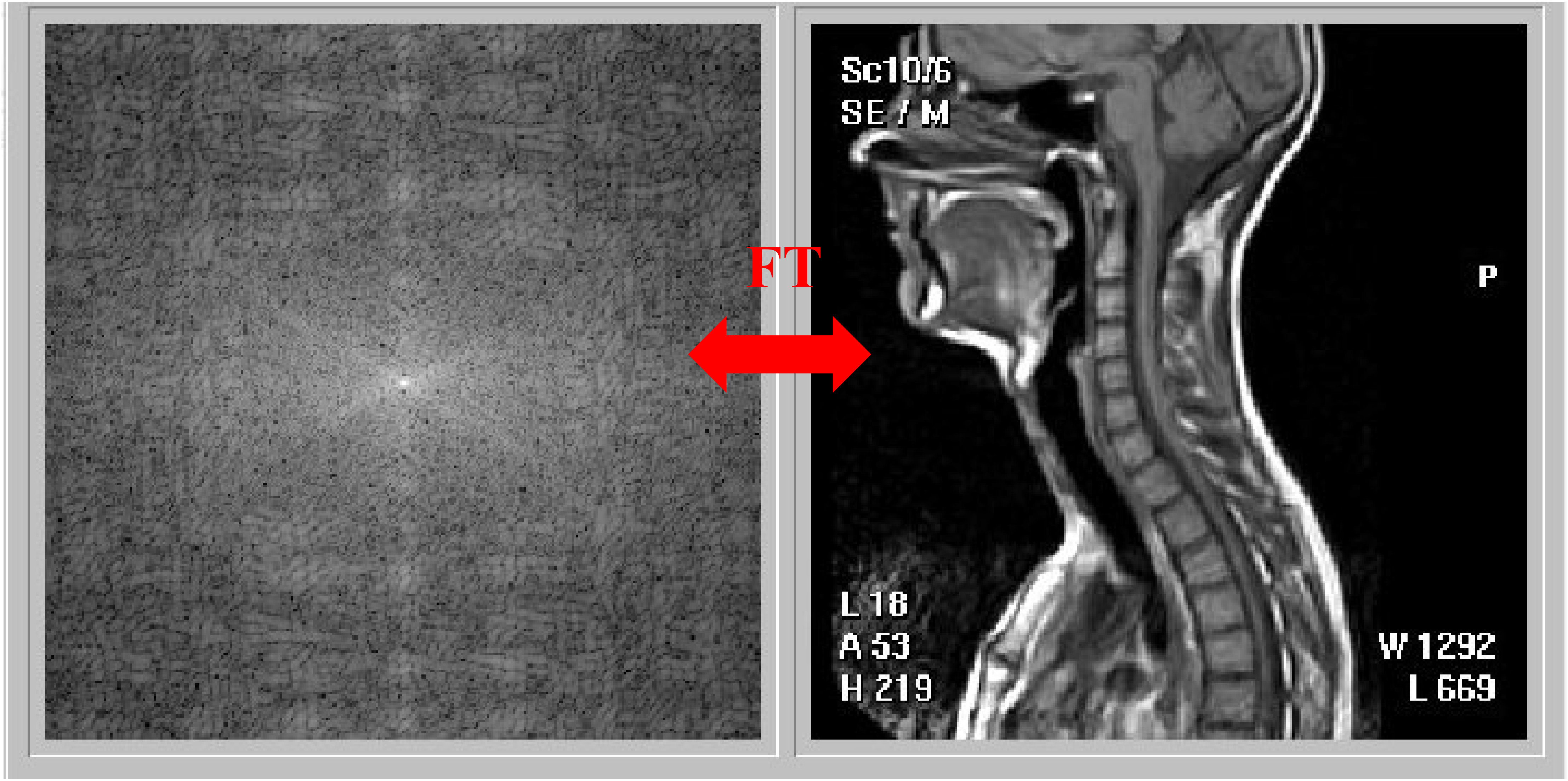


$\Delta k = 1 / FOV$,

$2\ k_{max} = 1 / \Delta x$

$S(k_x, k_y)$

$\rho(x,y)$



- How do we measure $S(\mathbf{k})$?

  We measure $S(t)$ with different values of $\mathbf{G}$

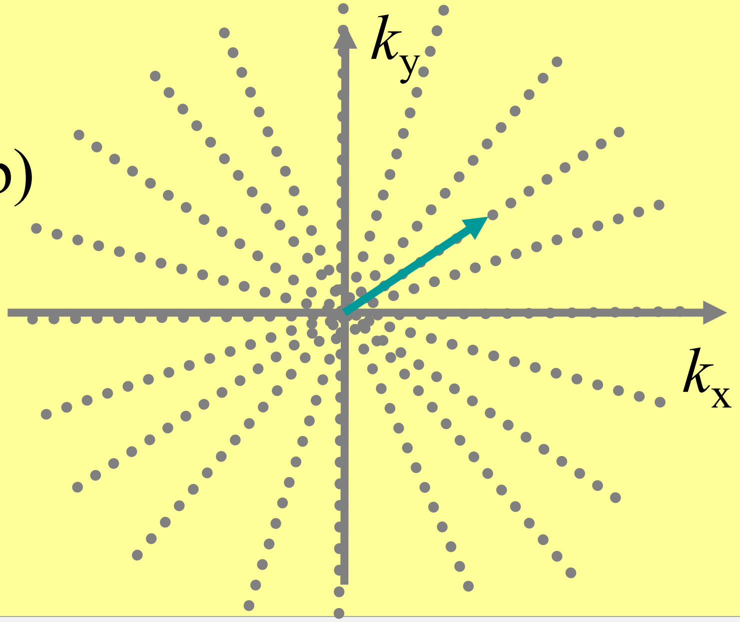
$$\mathbf{k} \equiv \frac{1}{2\pi}\gamma \mathbf{G} t$$

- Two famous examples of $S(\mathbf{k})$ sampling:

2D Projection reconstruction

a) Slice selection: k=(0,0,k_z)

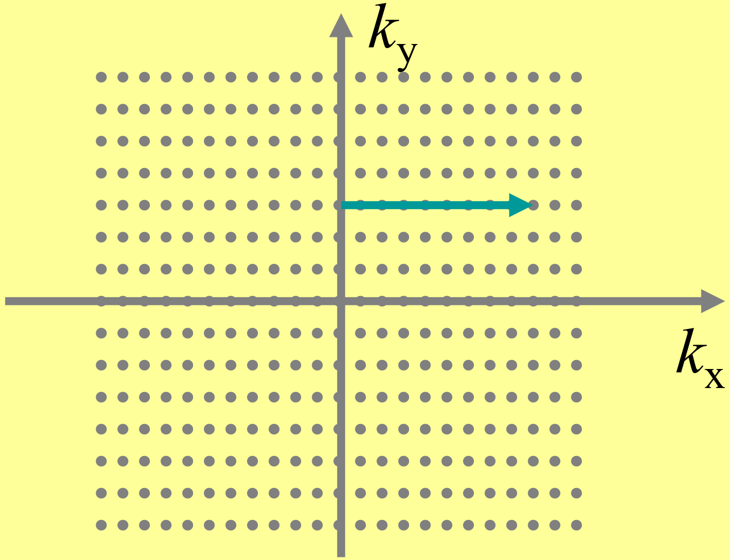
b)

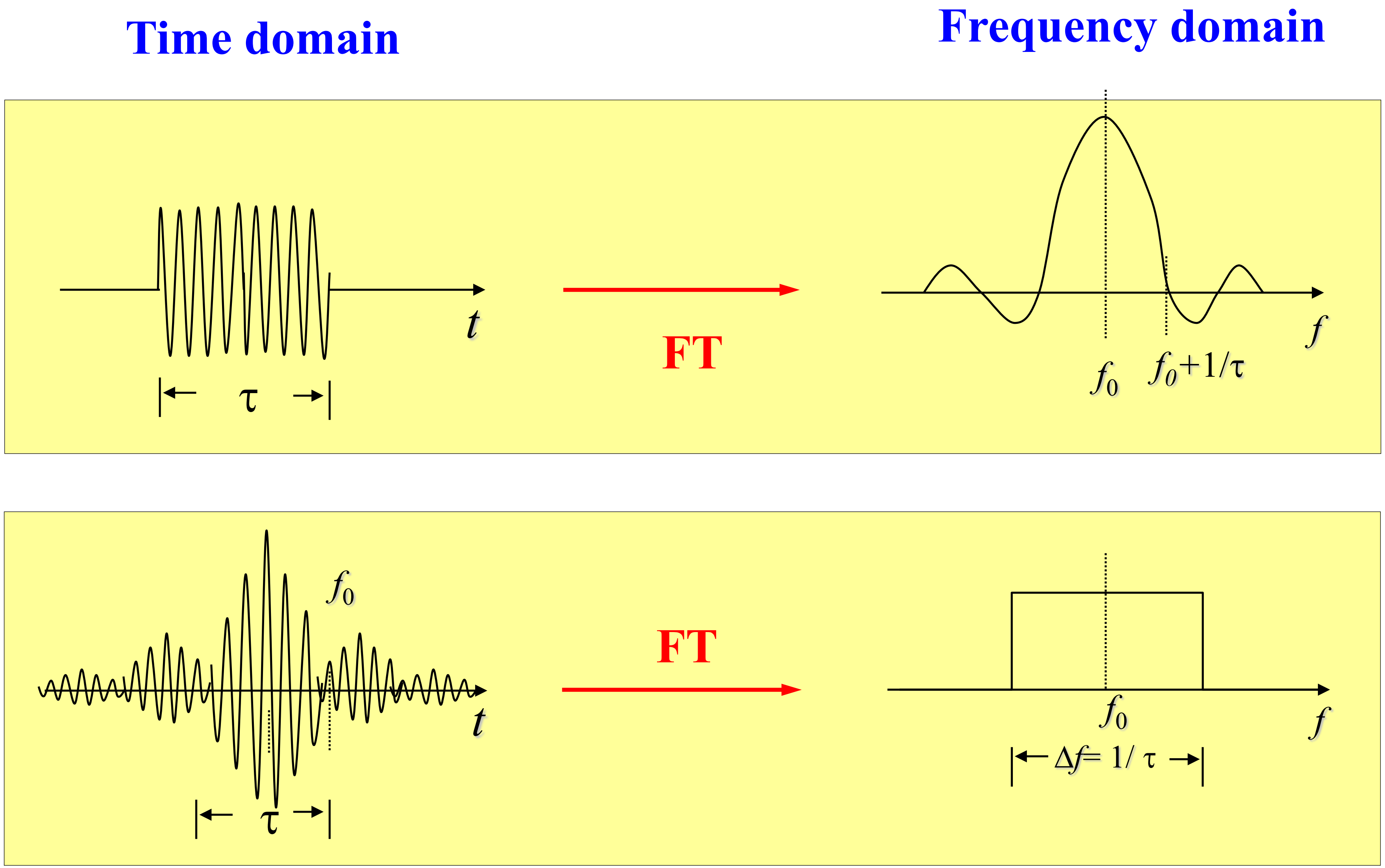


2D Fourier imaging

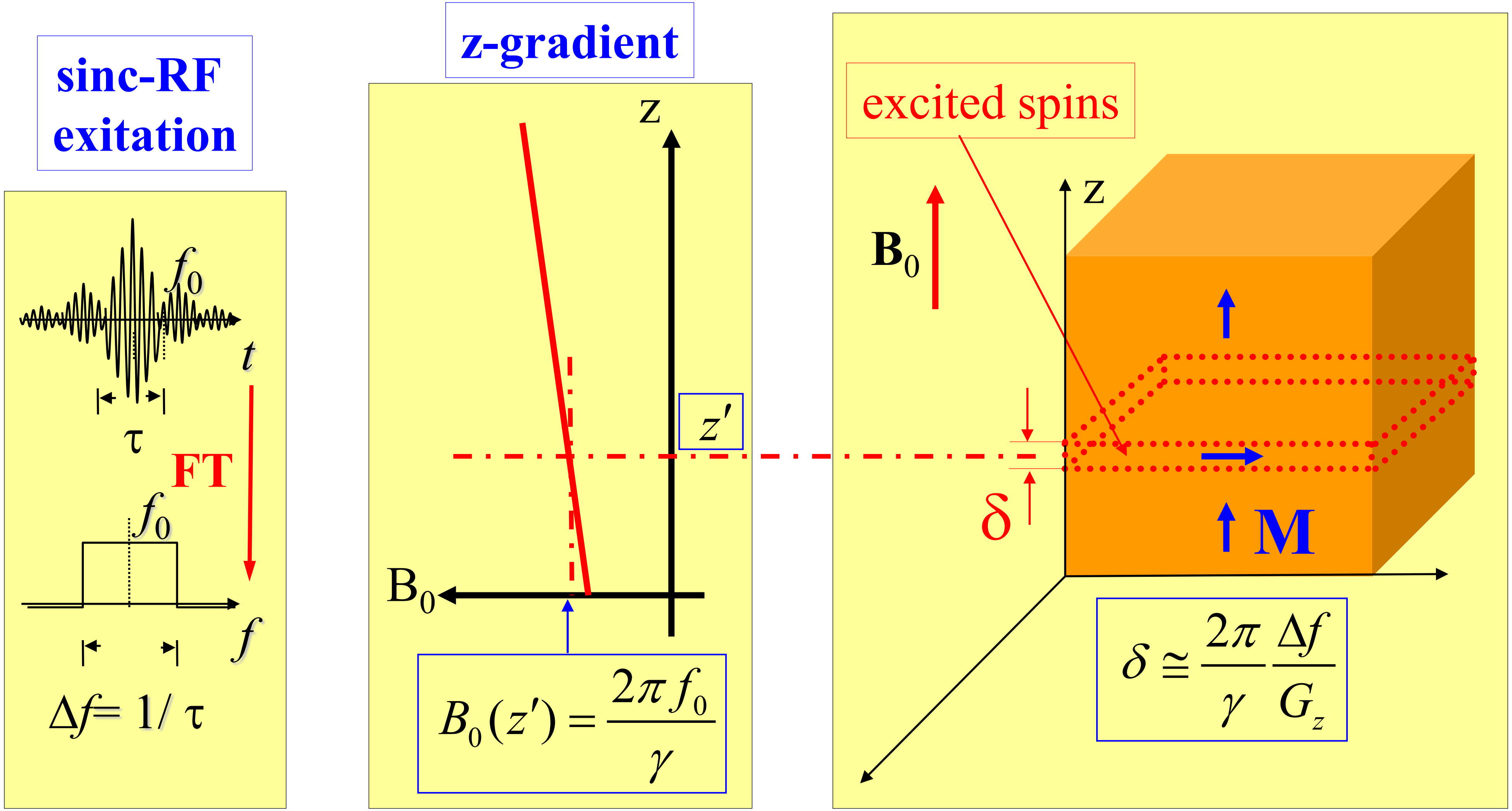
a) Slice selection: k=(0,0,k_z)

b)



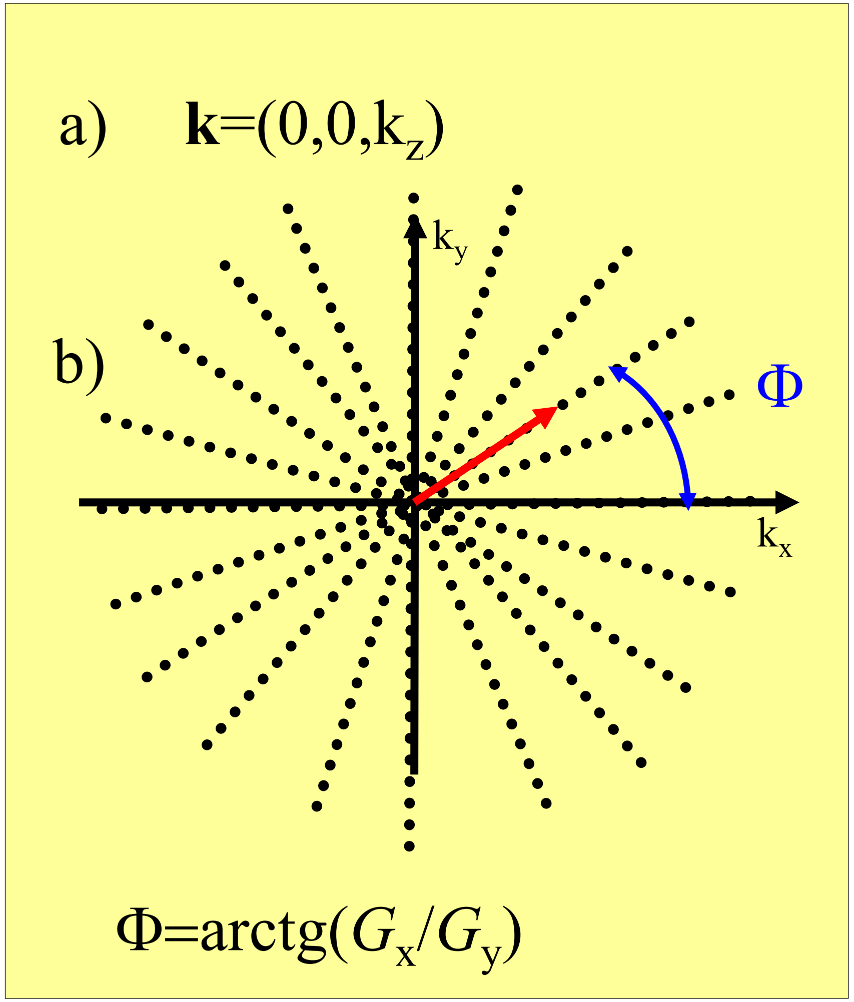

Time domain
Frequency domain
t
τ
FT
f
f_0
f_0+1/τ
f_0
t
τ
FT
f_0
f
Δf= 1/ τ

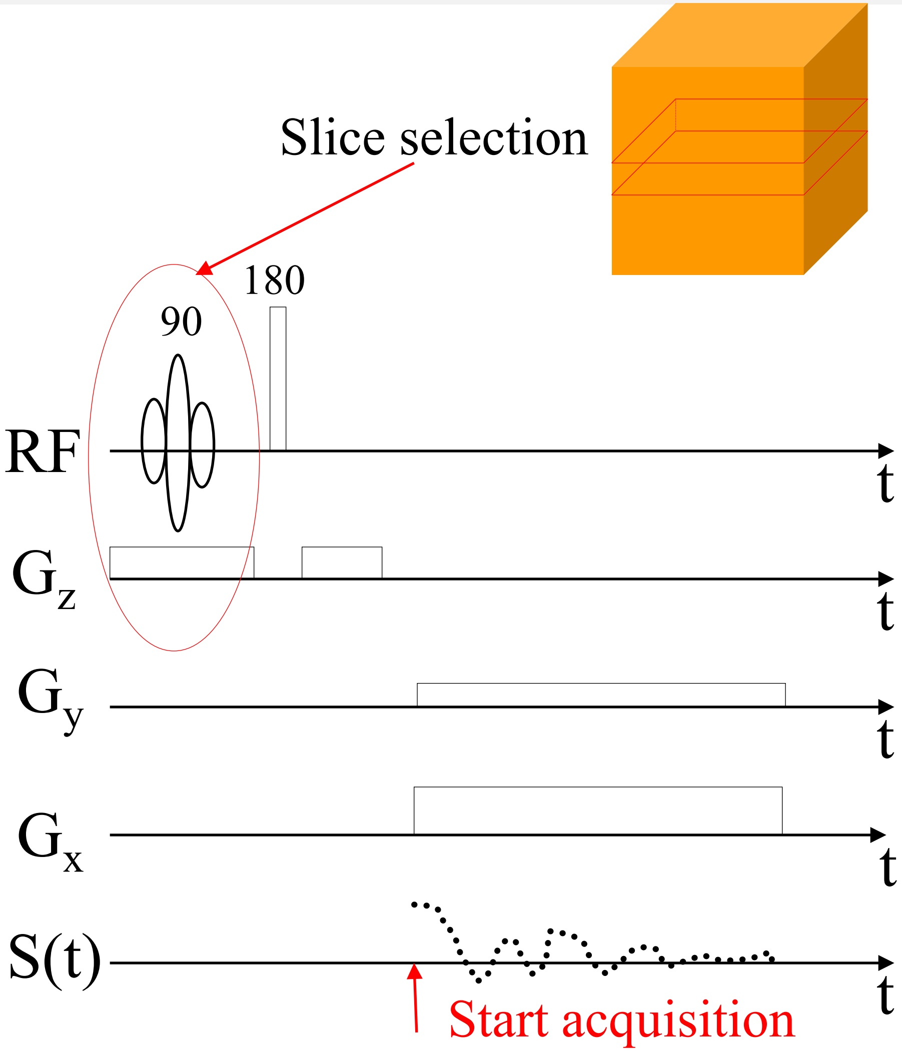
Slice selection



Only the spins in thin slice are excited (rotated)

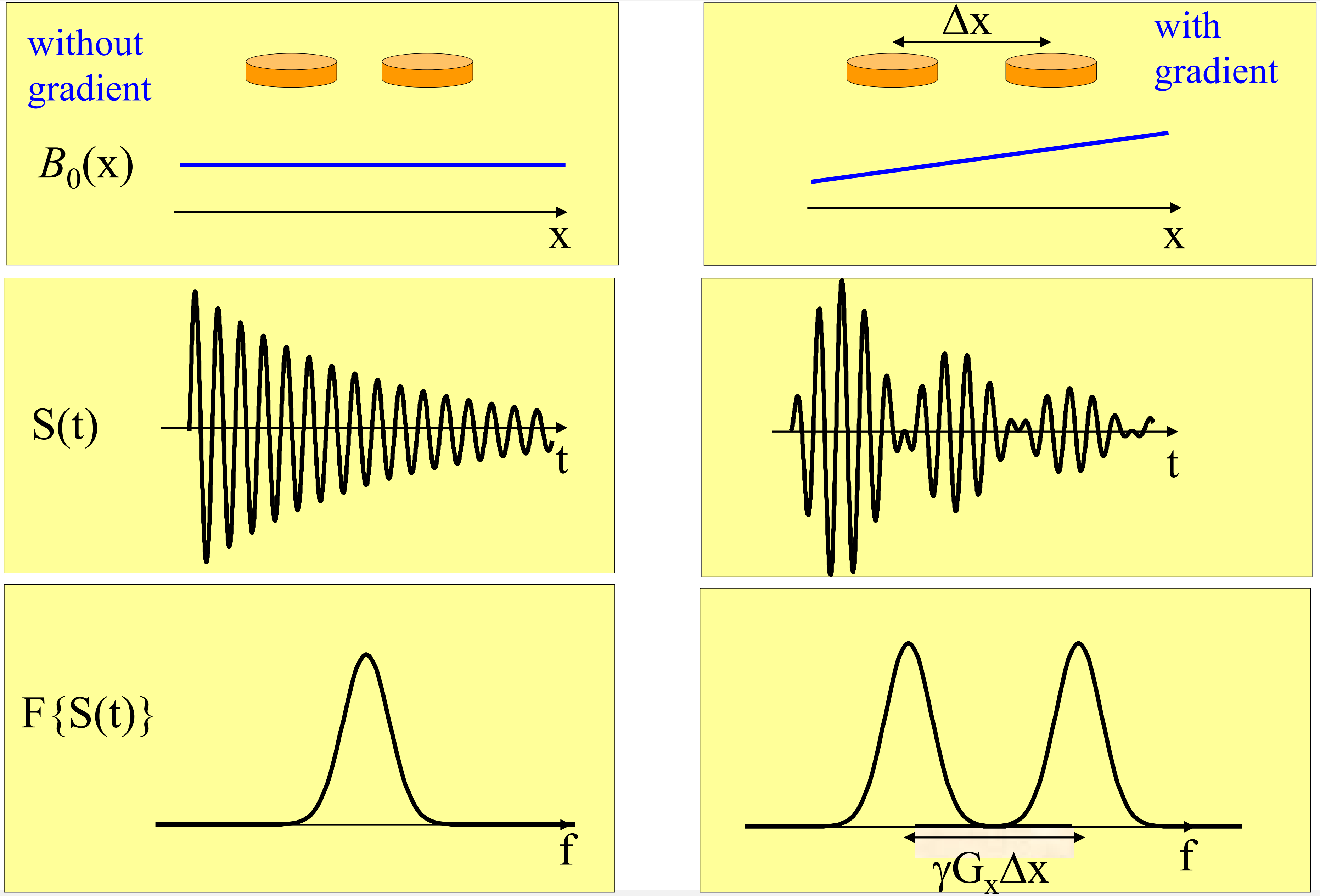
2D projection reconstruction





Sequence repeated with different G_x, G_y

2D projection reconstruction



Projection reconstruction (PR)

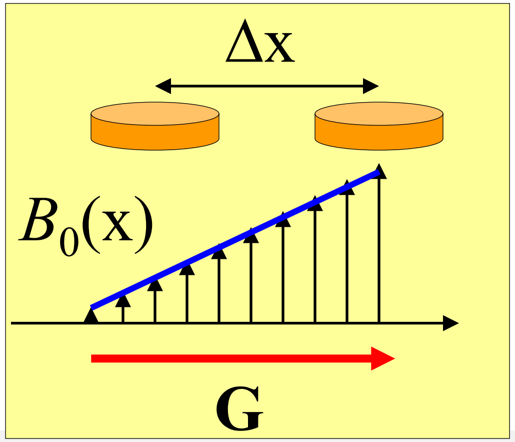
Suppose $\mathbf{G}=(G_x, 0, 0)$. Hence, $\mathbf{k}=(1/2\pi)\gamma G_x t, 0 , 0)$:

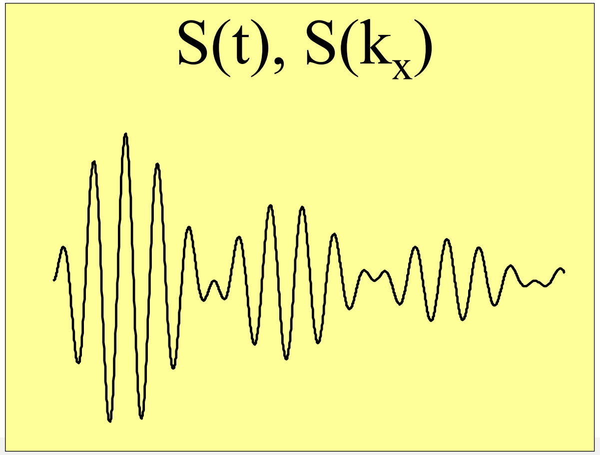
$$S(k_x) = \int\int\left(\int \rho(\mathbf{r})\exp(i2\pi k_x x)dx\right)dydz$$

and, consequently, its Fourier transform

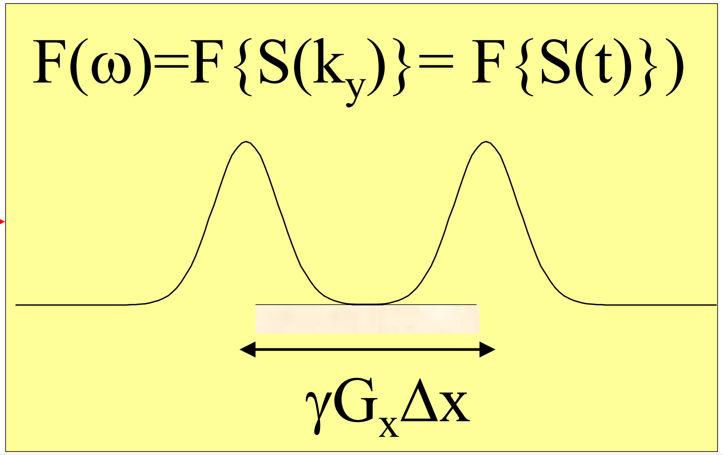
$$F\{S(k_x)\} = \int\int \rho(\mathbf{r})dydz$$

The NMR spectrum $F(\omega)$ corresponds to the projection of the spin density normal to the direction of the field gradient.

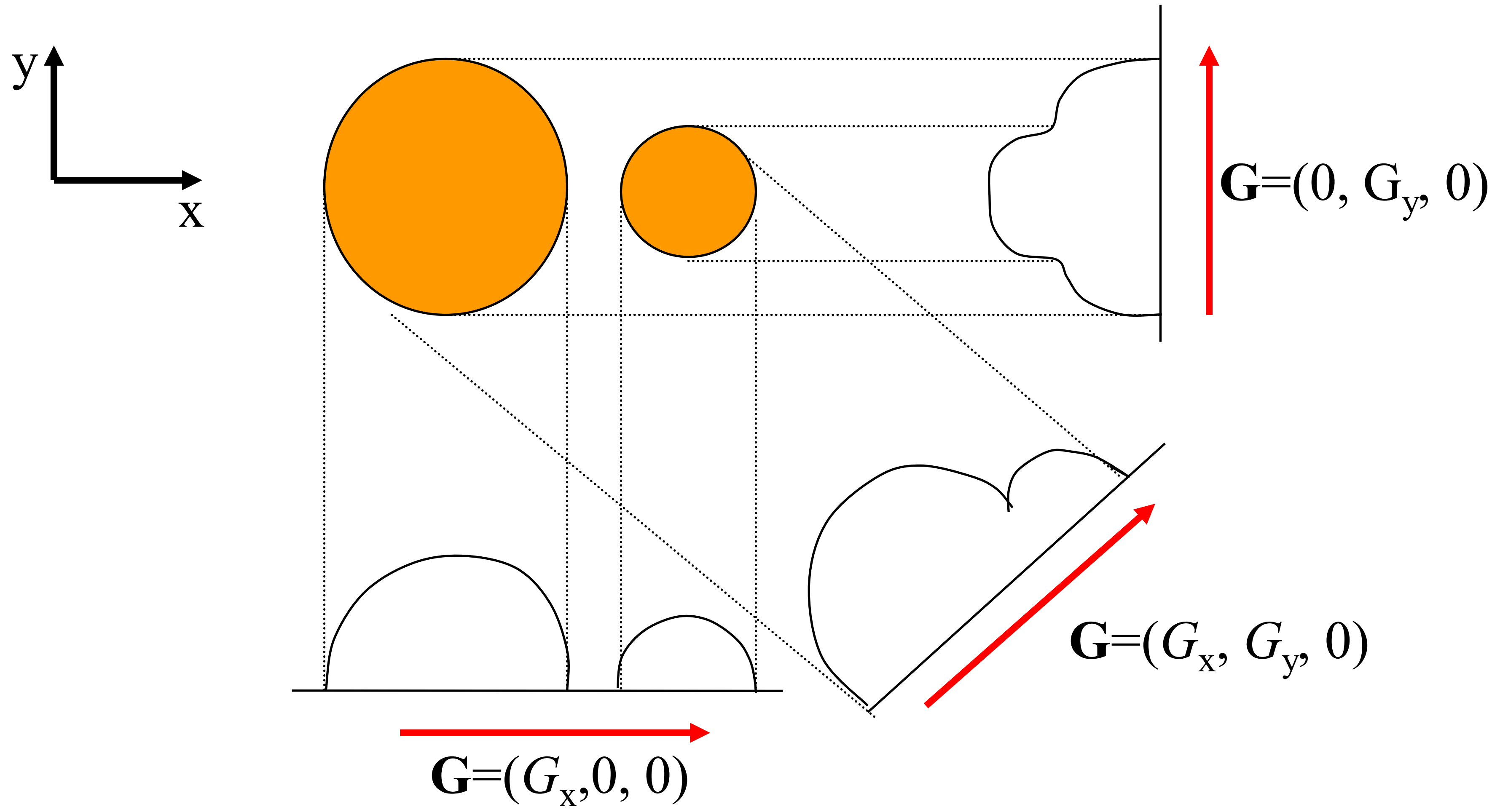




FT



Mapping of the xy-plane of the k-space:



MRI: T_1, T_2 image contrast

$$S(t) = \iiint \rho(\mathbf{r}) \exp(i\gamma \mathbf{G} \cdot \mathbf{r} t) d\mathbf{r}$$

More generally we have:

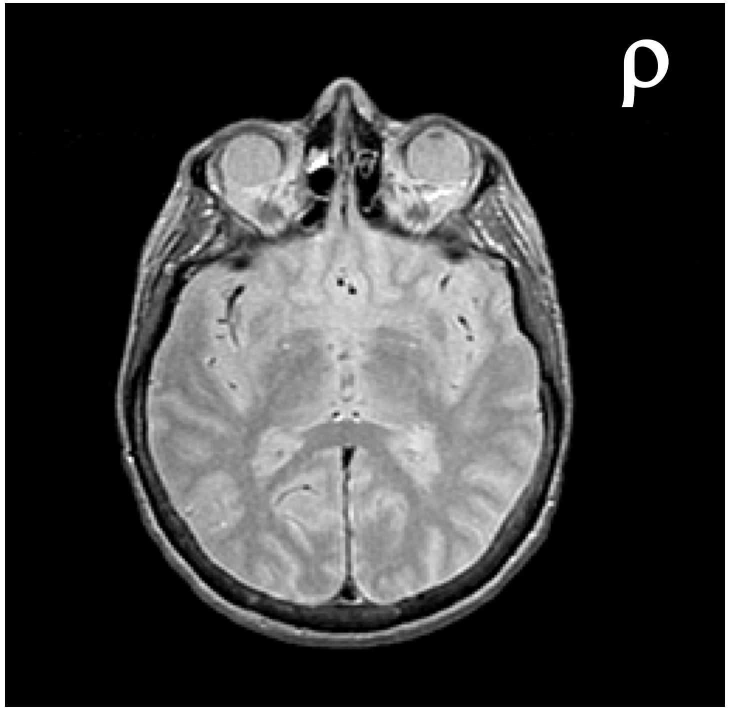
$$S(t) = \iiint C(\mathbf{r}) \rho(\mathbf{r}) \exp(i\gamma \mathbf{G} \cdot \mathbf{r} t) d\mathbf{r}$$

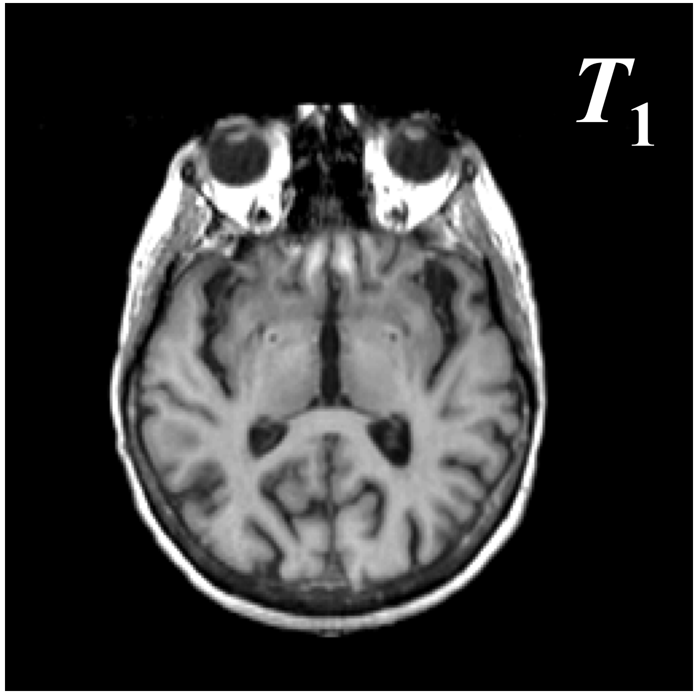
where :

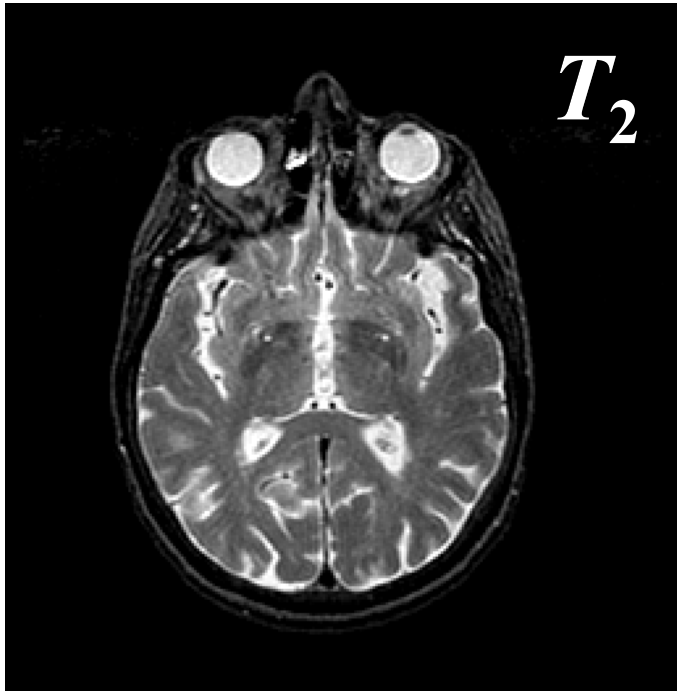
$$C(\mathbf{r}) = f(T_1(\mathbf{r}), T_2(\mathbf{r}), \ldots)$$

Tissue	T_1 (at 1.5 T) (ms)	T_2 (ms)
celebrospinal fluid	4500	2200
blood	1200	100-200
fat	260	80
liver	500	40
muscle	870	45
white matter	780	90
grey matter	900	100

The contrast in the images can be given by the spin density $\rho(\mathbf{r})$ but also by the relaxation times $T_1(\mathbf{r})$ and $T_2(\mathbf{r})$.


ρ


T_1


T_2

MRI: noise

Limiting factors for:
spatial resolution, **imaging time** and **image contrasts**

- Thermal noise of the series resistance of the detection coil (i.e., $(4kTR\Delta f)^{1/2}$)

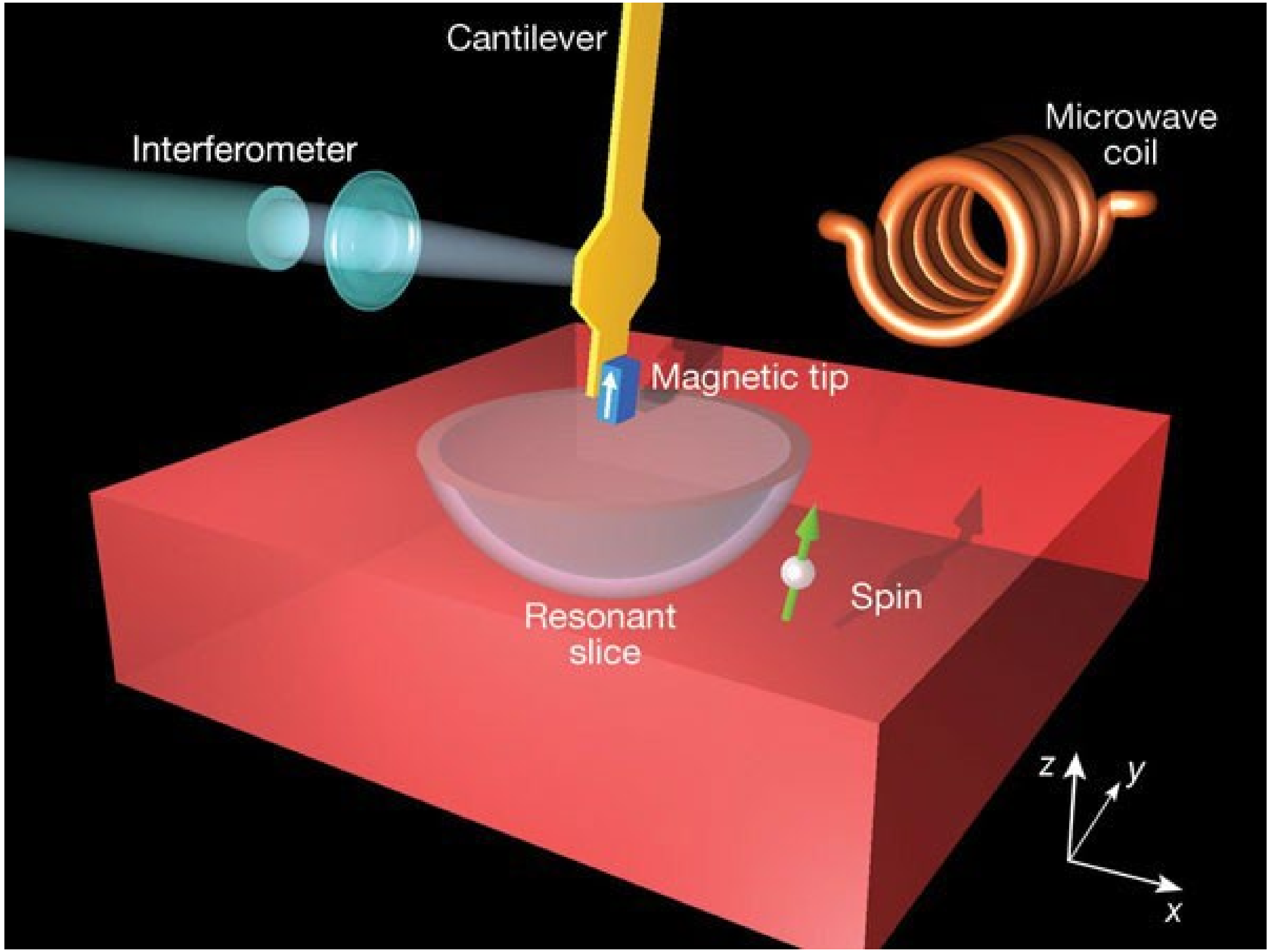
- Dielectric and inductive losses in the imaged object (for large objects)

- Diffusion

- Susceptibility artifacts

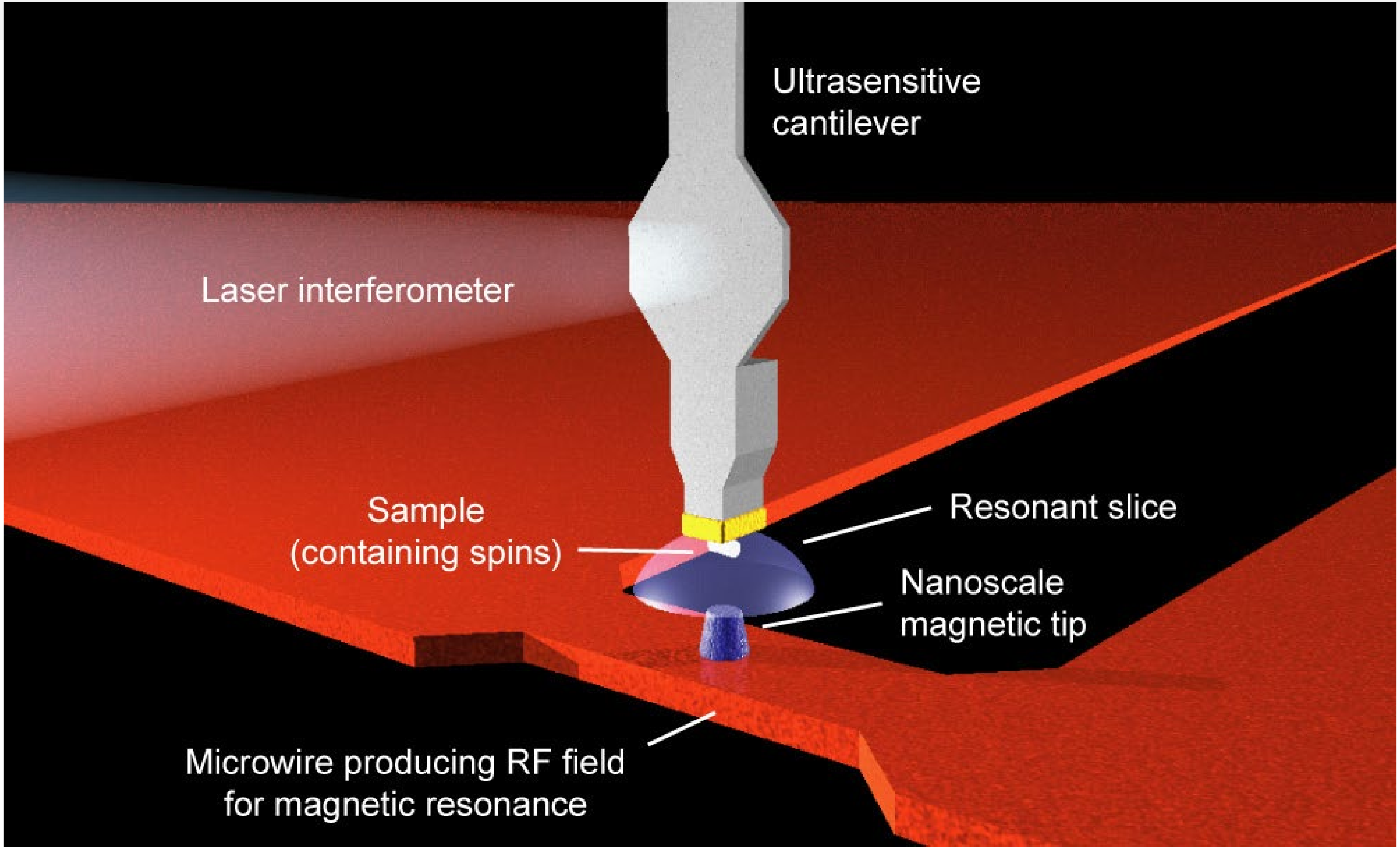
-….

Magnetic resonance force microscopy (MRFM)



A sort of MFM + MRI

Magnet on cantilever



Sample on cantilever

Other magnetic imaging techniques

- Magnetic particles (Bitter decoration): optical inspection of the arrangement of magnetic particles.

- Superconducting quantum interference devices (SQUID): magnetic flux dependent quantum interference

- Electron beam (Lorentz microscopy, Electron holography, SEMPA, SPLEEM): Lorentz force on the electrons, spin-polarization analysis of the emitted secondary electrons, spin-dependence of the quasi-elastic scattering of polarized electron from the surface.

- X-rays photon beam (XMCD): absorption dependent on the orientation of the local magnetization relative to the helicity of the x-rays.

- Light photon beam (magneto-optics): rotation of the polarization of light upon reflection (Kerr effect) or transmission (Faraday effect) caused by the sample magnetization.

- Scanning tunneling microscope (SP-STM): Tunneling probability between a magnetic sample and a magnetic tip (source of spin-polarized electrons) dependent on the magnetization of the sample.

Technique	Measured quantity	Quantitative/ Interpretability	Non-invasive	Apparatus Cost
Bitter decoration	Field gradient	Good in limited context	NO	<$100k
Lorenz microscopy	Field	Good in limited context	NO	$500k-$1M
Electron holography	Phase shifts due to field	Requires extensive modeling	NO	$1M
Magneto-optic imaging	Field	Good	NO/YES	<$100k
Magnetic force microscopy	First or second spatial derivative of field (integrated over tip area)	Poor	YES	$100k
Scanning SQUID microscopy	Flux through pickup loop	Good	YES	$100k
Scanning Hall probe microscopy	Field	Good	YES	$100k

Table 1	Bitter	Magneto-optic	MFM	Lorentz	DPC	SEMPA	Holography	XMCD	TXMCD	SPLEEM
Principles of method										
Contrast Origin	grad $\mathbf{B}_{ext.}$	**M**	grad $\mathbf{B}_{ext.}$	**B**	**B**	**M**	**B**, Φ_B	**M**	**M**	**M**
Quantitative	No	Yes	No	Yes	Yes	Yes	Yes	Yes	Yes	Yes
Practical Aspects										
Best Resolution (nm)	100	300	40	~10	~2	20	~5	300	30	20
Typical Resolution (nm)	500	1000	100	50	20	200	20	500	60	40
Information Depth (nm)	500	20	20 - 500	Sample Thickness	Sample Thickness	2	Sample Thickness	2 - 20	Sample Thickness	1
Acquisition Time	.03 sec.	10^{-8} - 1 sec.	5 - 30 minutes	0.04-30 sec.	5-50 sec.	1 - 100 minutes	.03-10 sec.	.03 sec. to 10 minutes	3 sec	1 sec
Insulators	Yes	Yes	Yes	No	No	No	No	Yes	Yes	No
Vacuum Requirement	None	None	None	HV	HV	UHV	HV	UHV	None	UHV

EPFL

Table 1	Bitter	Magneto-optic	MFM	Lorentz	DPC	SEMPA	Holography	XMCD	TXMCD	SPLEEM
Complexity	Low	Moderate	Moderate	Moderate	Mod./High	High	High	High	High	High
Commercially Available	Yes	Yes	Yes	Yes	No	No	Yes	No	No	No
Cost	1 K$	50-500 K$	150 K$	0.2-1 M$	1 M$	800 K$	1.3 M$	300+ K$	300+ K$	1 M$
Sample Prep										
Sample thickness (nm)	No Limit	No Limit	No Limit	<150	<150	No Limit	<150	No Limit	< 100	No Limit
Special Smoothness	Yes	Yes	Yes	Yes	Yes	No	Yes	No	Yes	Yes
Clean Surface Required	No	No	No	No	No	Yes	No	No	No	Yes
Specimen Modification										
Maximum Applied External Field (kA/m)	No Limit	No Limit	800	500 (vert.) 100 (horiz.)	500 (vert.) 100 (horiz.)	None	100	None	No Limit	None
Problems										
Topographic Feedthrough	Yes	Yes	Yes	Some	Some	No	Some	No	No	No
Crystallographic	No	No	No	Yes	Yes	No	Yes	No	Not	Yes